Induction Motor

Induction Motors

- The single-phase induction motor is the most frequently used motor in the world
- Most appliances, such as washing machines and refrigerators, use a single-phase induction machine
- Highly reliable and economical

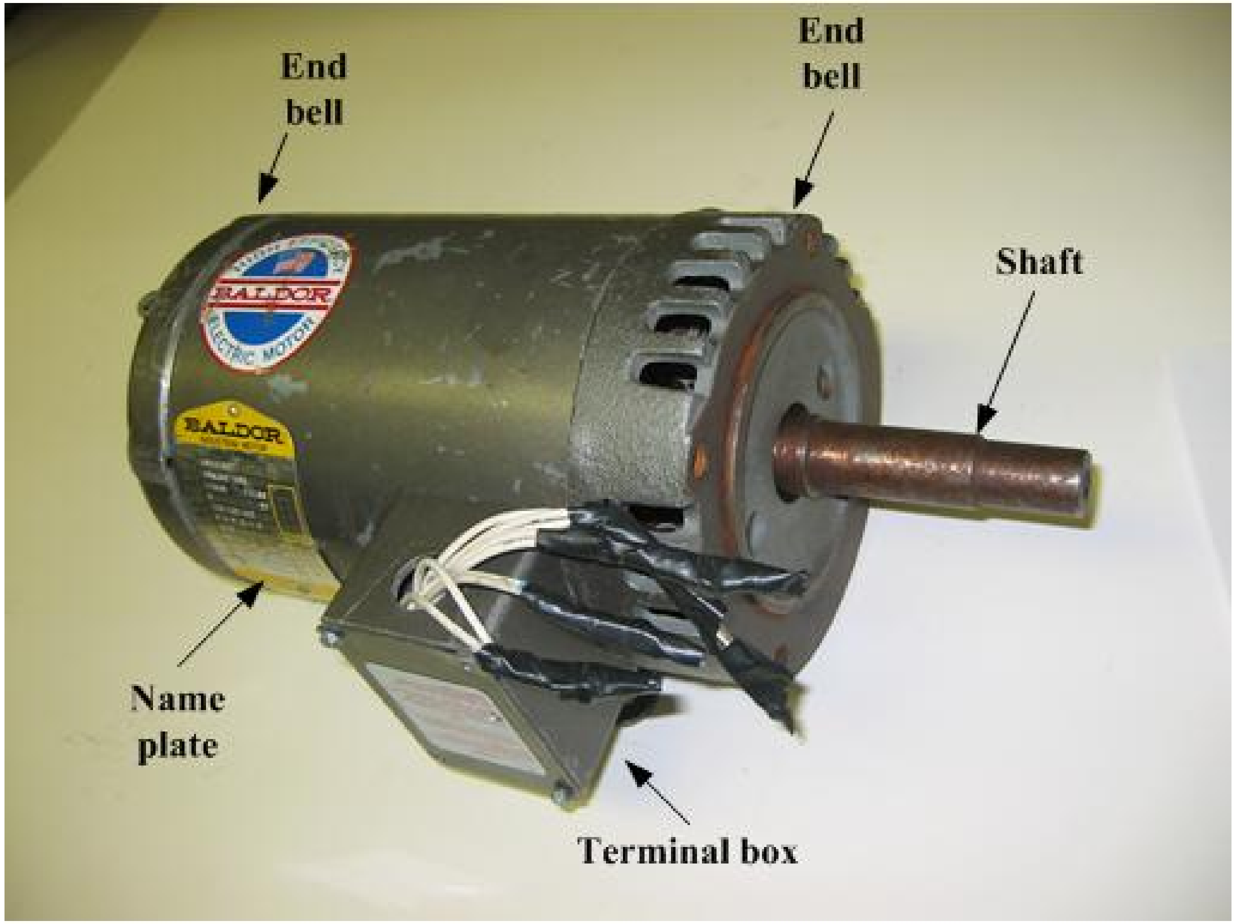


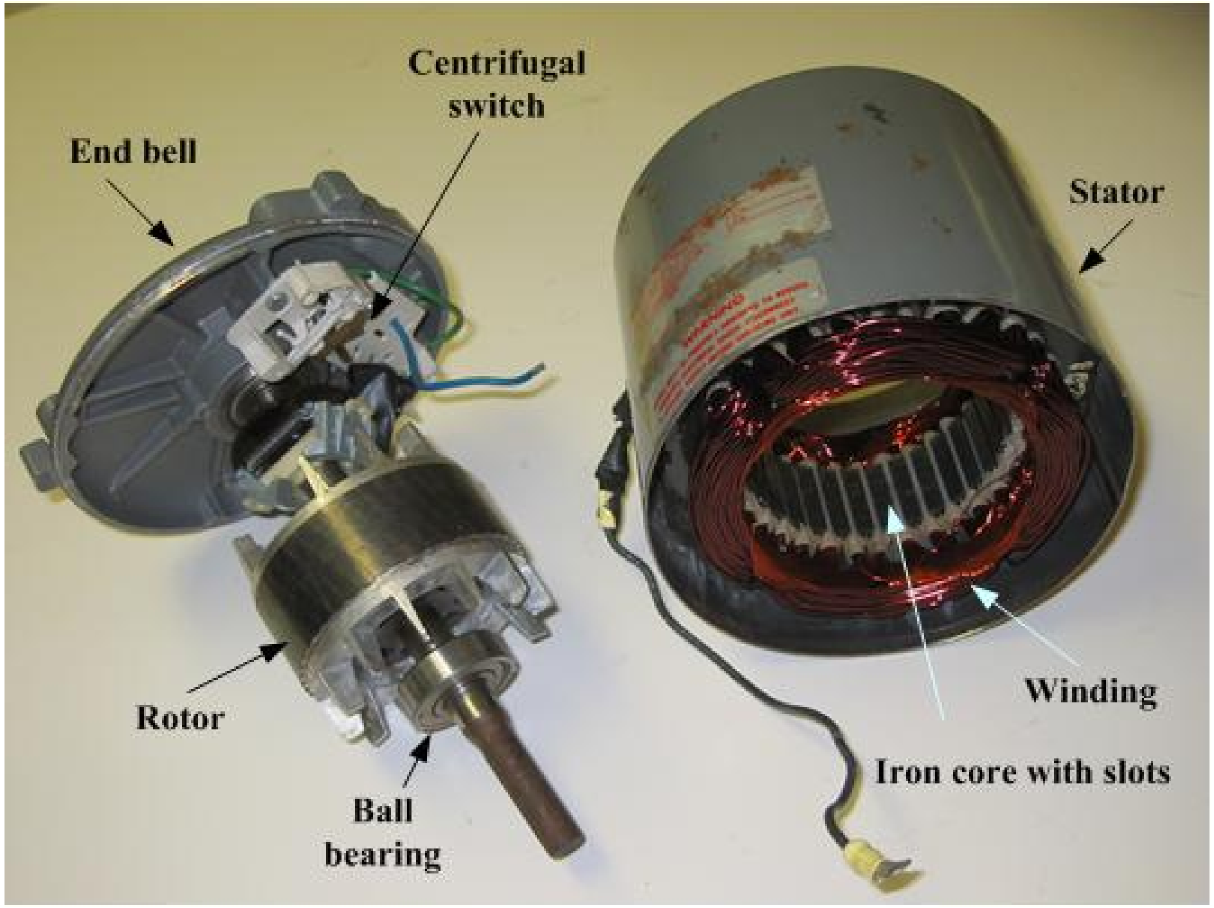
Figure 1 Single-phase induction motor.

Induction Motors

- For industrial applications, the three-phase induction motor is used to drive machines
- Figure 2 Large three-phase induction motor. (Courtesy Siemens).

Induction Motors

Figure 3
Induction motor components.

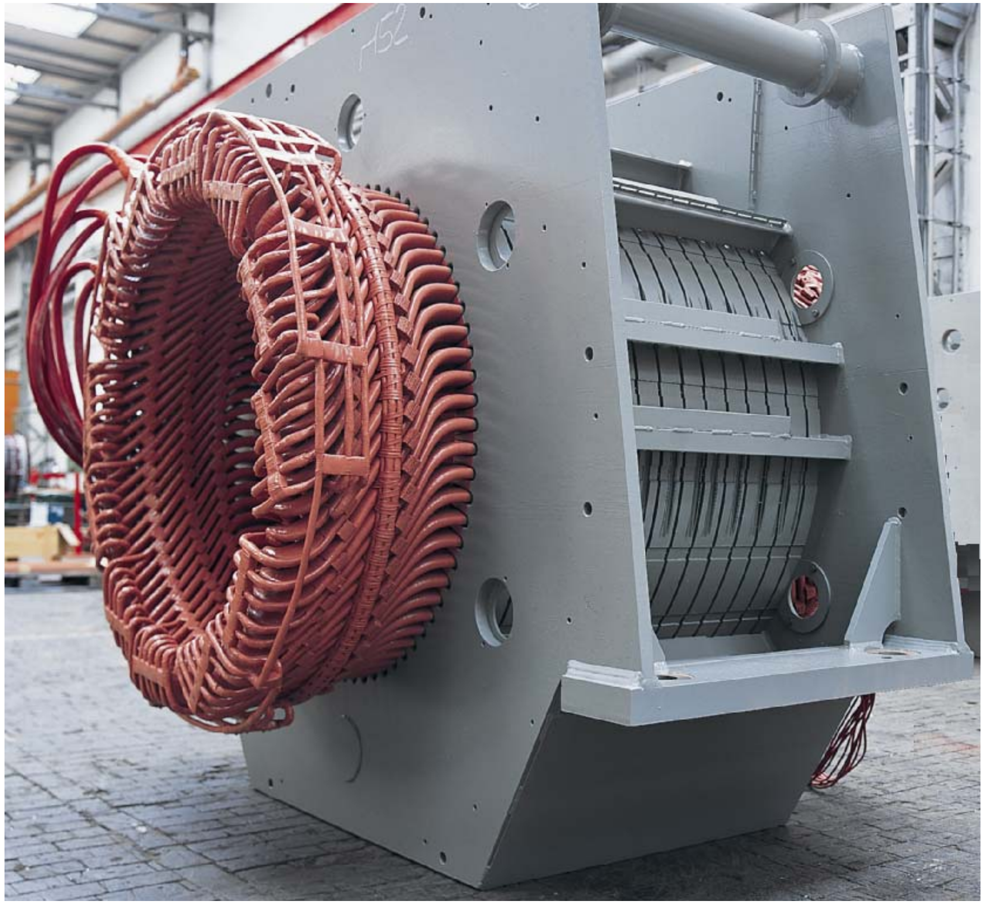


Induction Motors

- The motor housing consists of three parts:
  - The cylindrical middle piece that holds the stator iron core,
  - The two bell-shaped end covers holding the ball bearings.
  - This motor housing is made of cast aluminum or cast iron. Long screws hold the three parts together.
  - The legs at the middle section permit the attachment of the motor to a base.
  - A cooling fan is attached to the shaft at the left-hand side. This fan blows air over the ribbed stator frame.

Induction Motors

Figure 4 Stator of a large induction motor. (Courtesy Siemens).



Induction Motors

- The iron core has cylindrical shape and is laminated with slots
- The iron core on the figure has paper liner insulation placed in some of the slots.
- In a three-phase motor, the three phase windings are placed in the slots
- A single-phase motor has two windings: the main and the starting windings.
- Typically, thin enamel insulated wires are used

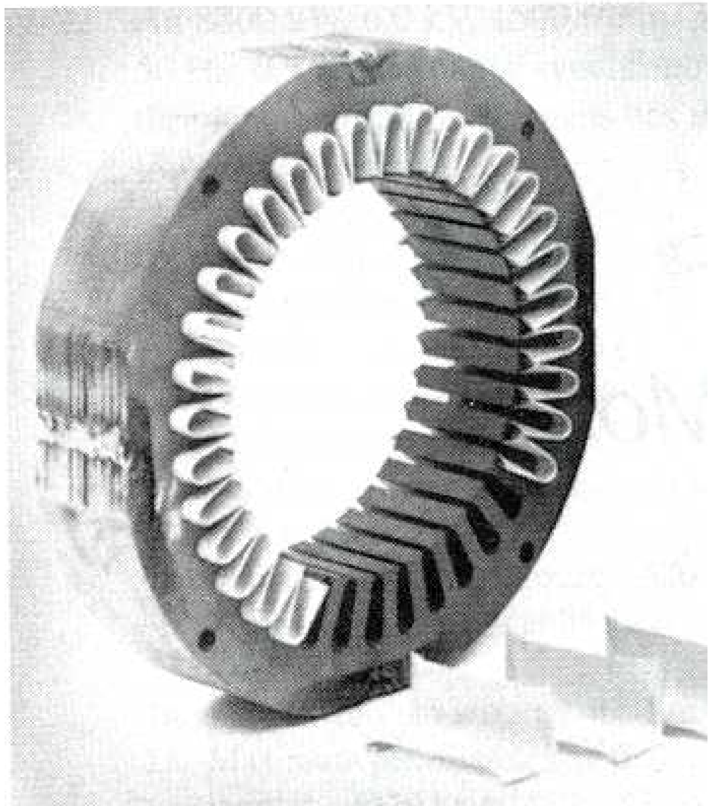


Figure 5 Stator iron core without windings

Induction Motors

- A single-phase motor has two windings: the main and the starting windings
- The elements of the laminated iron core are punched from a silicon iron sheet.
- The sheet has 36 slots and 4 holes for the assembly of the iron core.

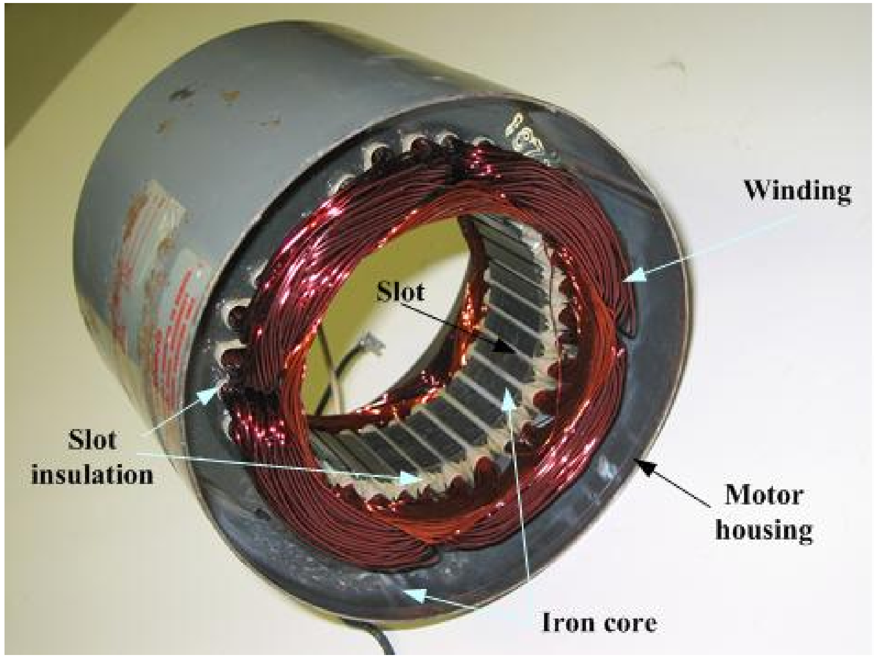


Figure 6 Single-phase stator with main windings.

Induction Motors

- The elements of the laminated iron core are punched from a silicon iron sheet.

- The sheet has 36 slots and 4 holes for the assembly of the iron core

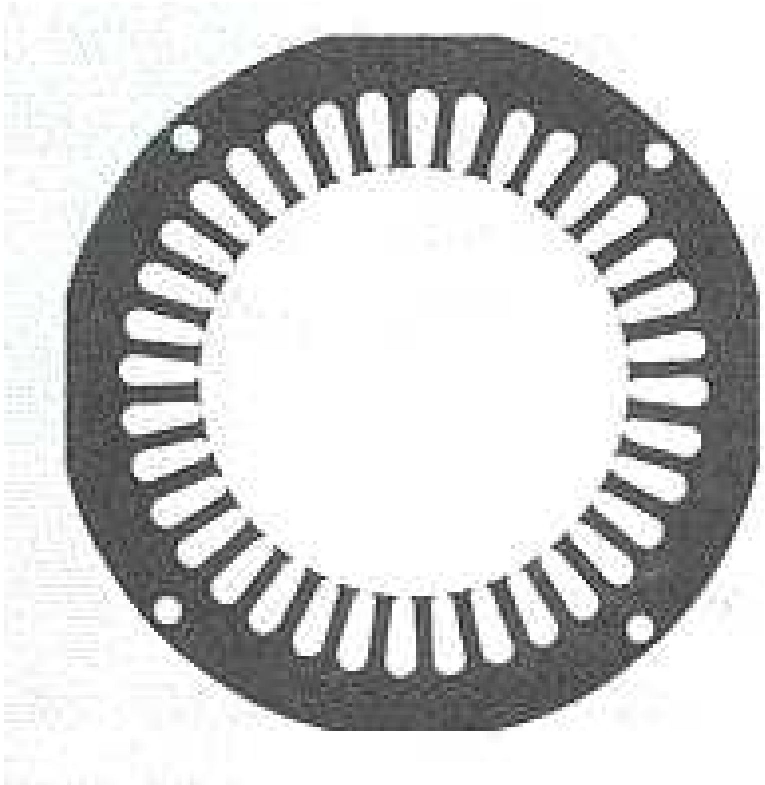
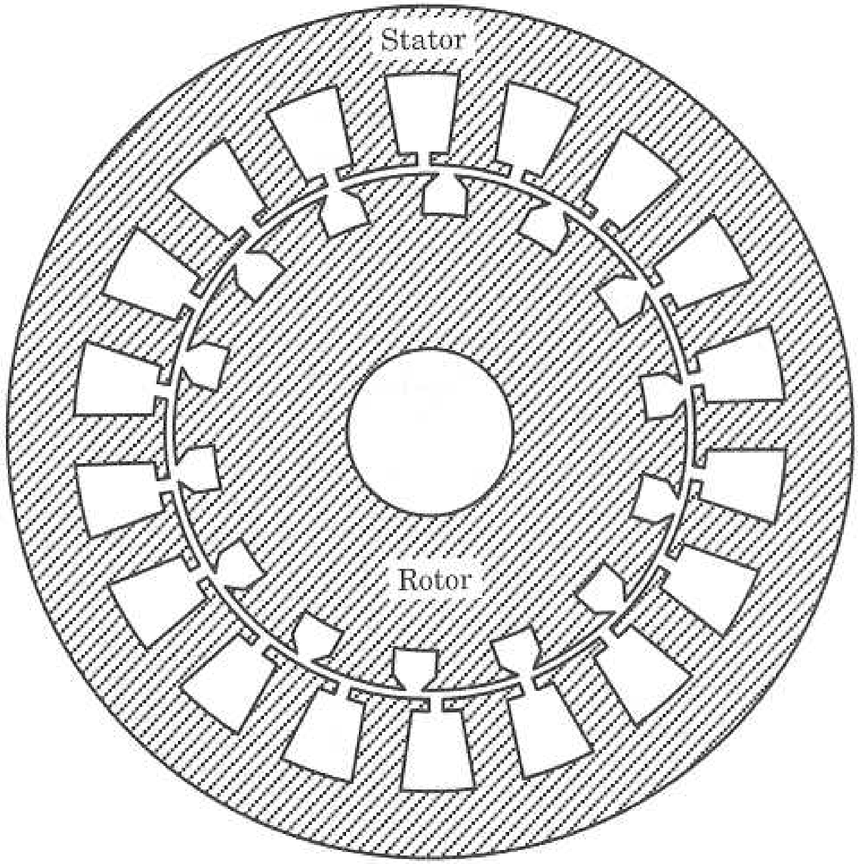


Figure 7 Stator iron core sheet.

Induction Motors

Figure 8 Stator and rotor magnetic circuit



Induction Motors

Squirrel cage rotor.

- **This rotor has a laminated iron core with slots, and is mounted on a shaft.**
- **Aluminum bars are molded in the slots and the bars are short circuited with two end rings.**
- **The bars are slanted on a small rotor to reduce audible noise.**
- **Fins are placed on the ring that shorts the bars. These fins work as a fan and improve cooling.**

Induction Motors

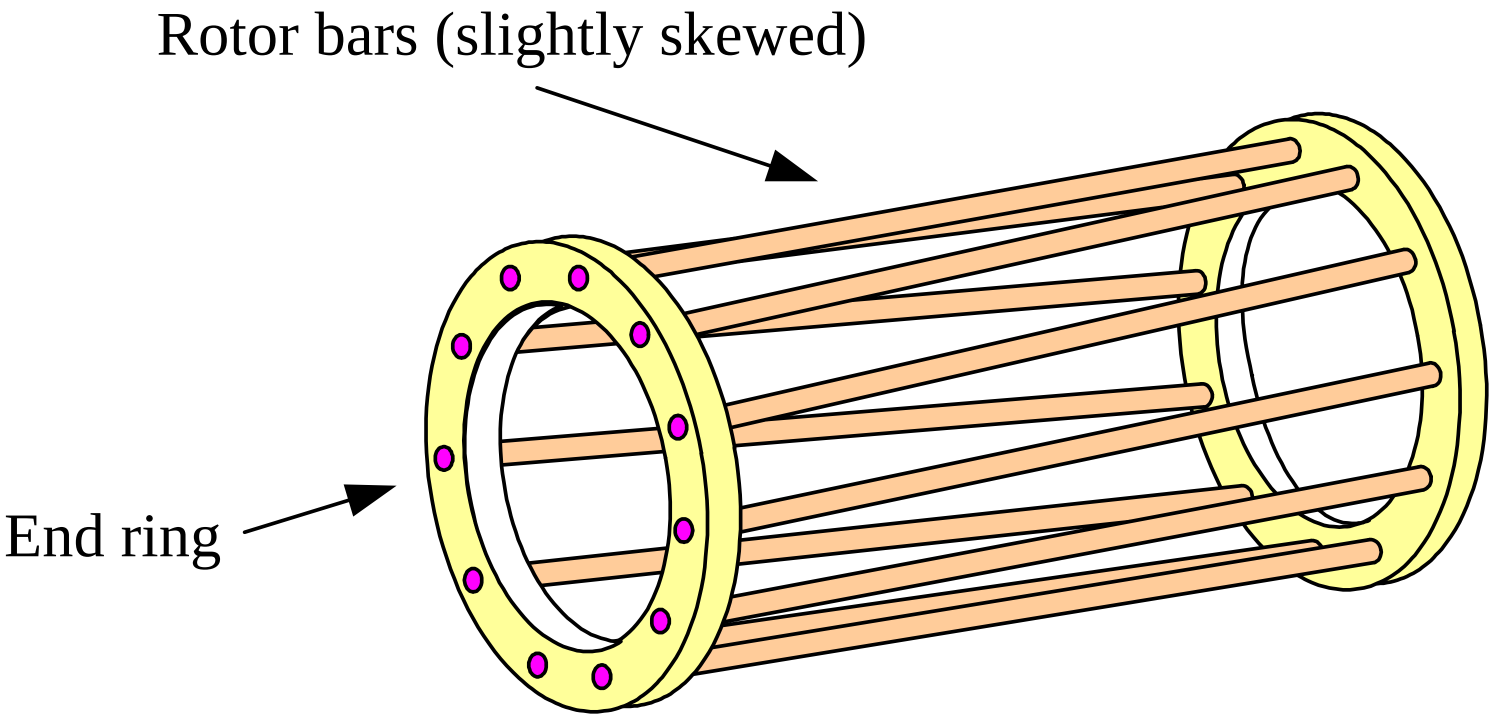


Figure 9 Squirrel cage rotor concept.

Induction Motors

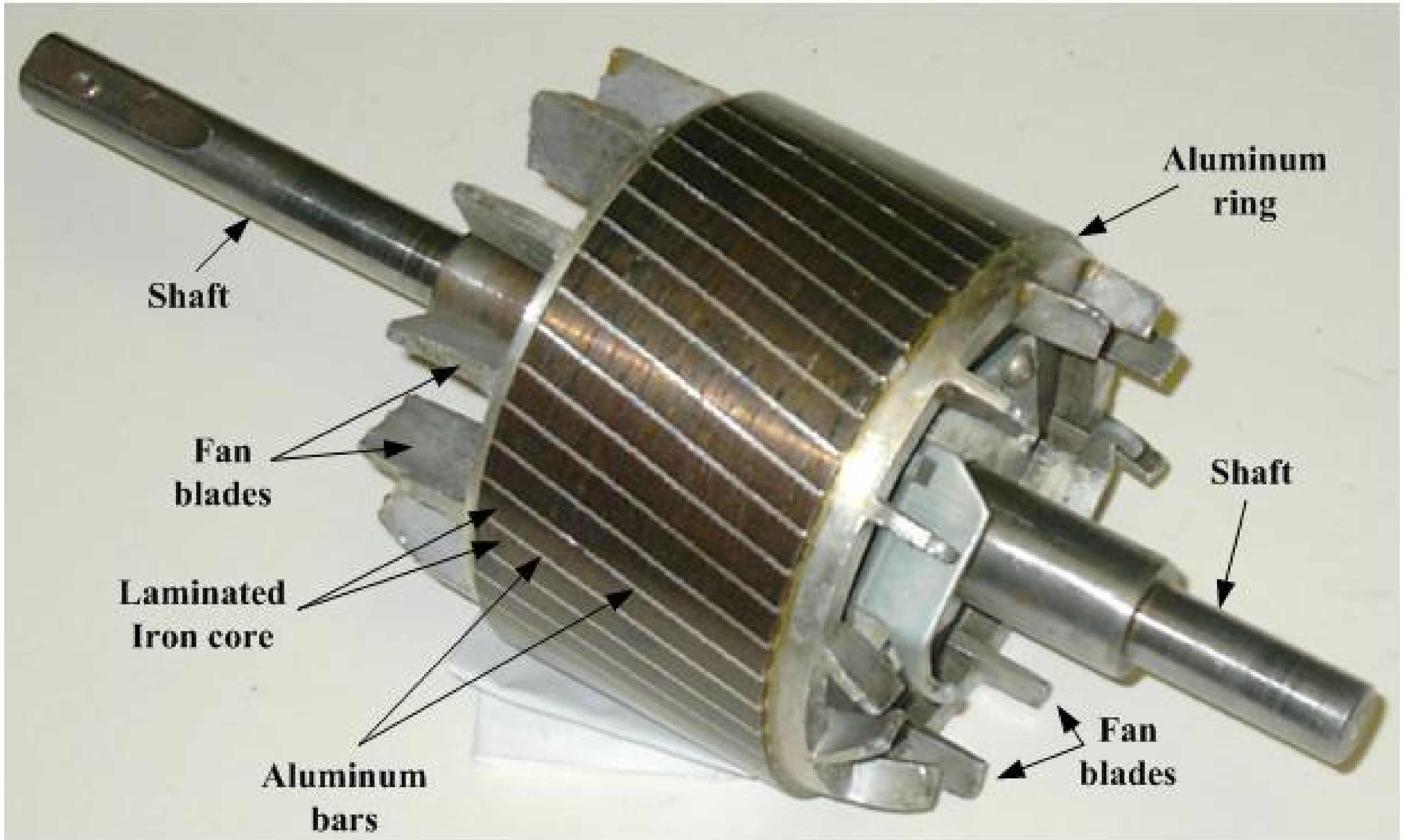


Figure 10 Squirrel cage rotor.

Induction Motors

Wound rotor.

- Most motors use the squirrel-cage rotor because of the robust and maintenance-free construction.
- However, large, older motors use a wound rotor with three phase windings placed in the rotor slots.
- The windings are connected in a three-wire wye.
- The ends of the windings are connected to three slip rings.
- Resistors or power supplies are connected to the slip rings through brushes for reduction of starting current and speed control

Induction Motors

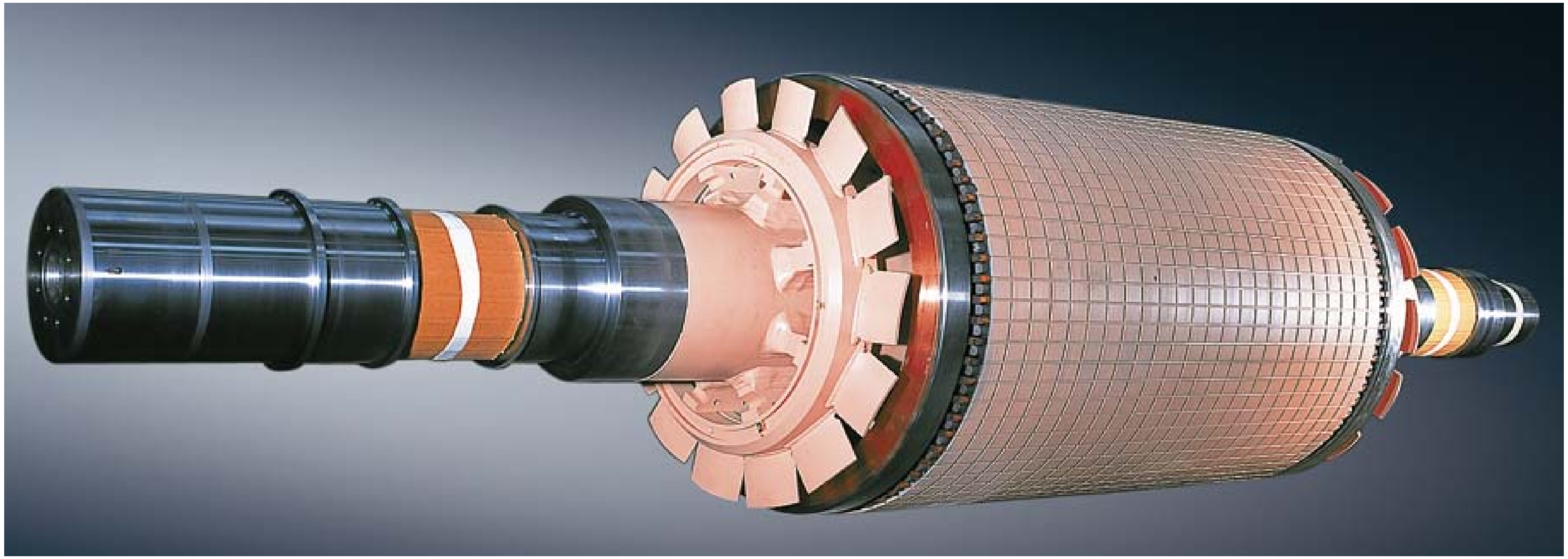


Figure 11 Rotor of a large induction motor. (Courtesy Siemens).

Operating principle

Induction Motors

- This two-pole motor has three stator phase windings, connected in three-wire wye.
- Each phase has 2 × 3 = 6 slots. The phases are shifted by 120°
- The squirrel cage rotor has short-circuited bars.
- The motor is supplied by balanced three-phase voltage at the terminals.
- The stator three-phase windings can also be connected in a delta configuration.

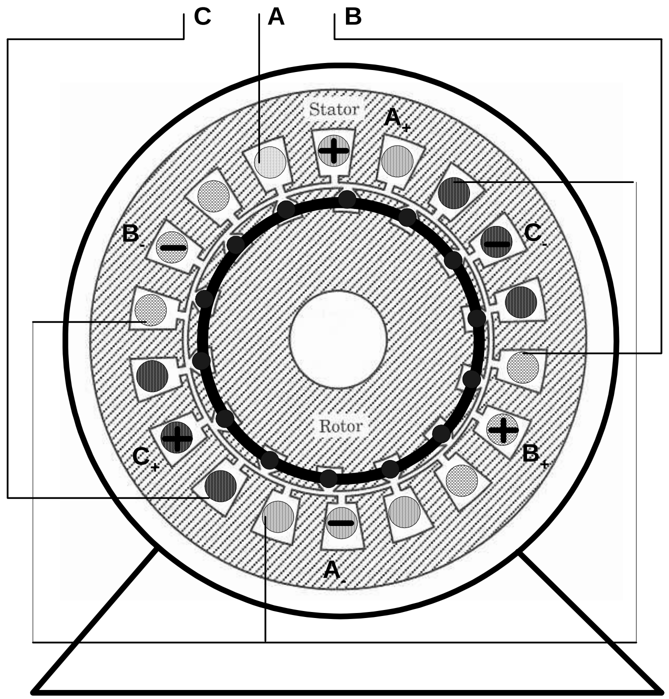


Figure 11 Connection diagram of a two-pole induction motor with squirrel cage rotor.

Induction Motors

Operation Principle

- The three-phase stator is supplied by balanced three-phase voltage that drives an ac magnetizing current through each phase winding.
- The magnetizing current in each phase generates a pulsating ac flux.
- The flux amplitude varies sinusoidally and the direction of the flux is perpendicular to the phase winding.

Induction Motors

Operation Principle

- The three-phase stator is supplied by balanced three-phase voltage that drives an ac magnetizing current through each phase winding.
- The magnetizing current in each phase generates a pulsating ac flux.
- The total flux in the machine is the sum of the three fluxes.
- The summation of the three ac fluxes results in a rotating flux, which turns with constant speed and has constant amplitude.

Induction Motors

Operation Principle

- The rotating flux induces a voltage in the short-circuited bars of the rotor. This voltage drives current through the bars.
- The induced voltage is proportional with the difference of motor and synchronous speed. Consequently the motor speed is less than the synchronous speed
- The interaction of the rotating flux and the rotor current generates a force that drives the motor.
- The force is proportional with the flux density and the rotor bar current

Induction Motors

- The figure shows the three components of the magnetic field at a phase angle of –60°.
- Each phase generates a magnetic field vector.
- The vector sum of the component vectors $\mathbf{\Phi_a}$, $\mathbf{\Phi_b}$, $\mathbf{\Phi_c}$ gives the resulting rotating field vector $\mathbf{\Phi_{rot}}$,
- The amplitude is 1.5 times the individual phase vector amplitudes, and $\mathbf{\Phi_{rot}}$ rotates with constant speed.

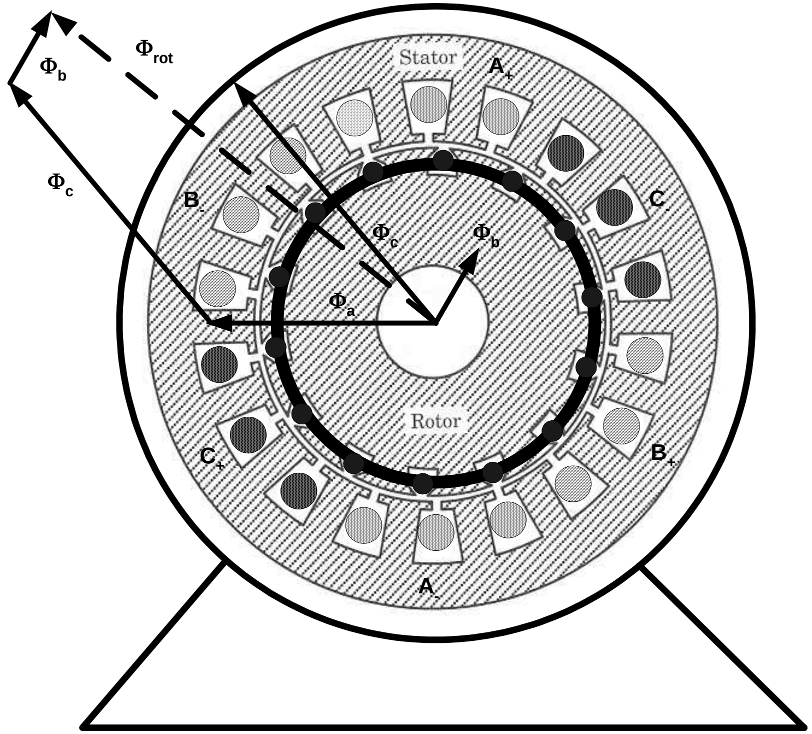


Figure 12 Three-phase winding-generated rotating magnetic field.

Induced Voltage Generation

Induction Motors

Faraday's law

- **Voltage is induced in a conductor that moves perpendicular to a magnetic field,**
- **The induced voltage is:**

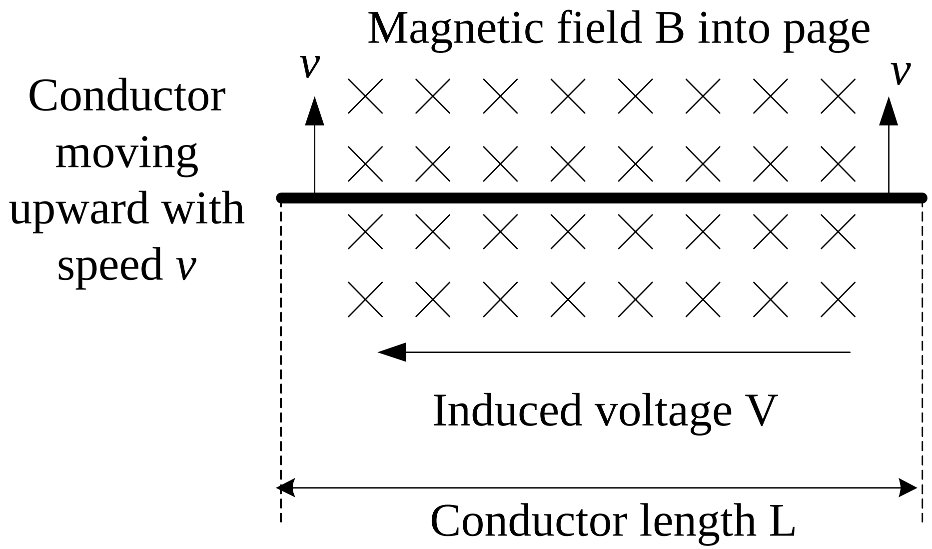
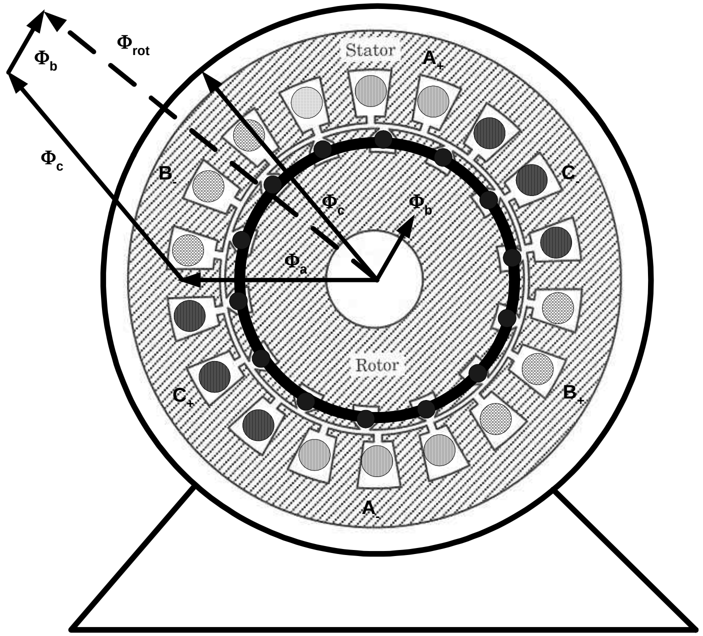


Figure 14 Voltage induced in a conductor moving through a magnetic field.

Induction Motors

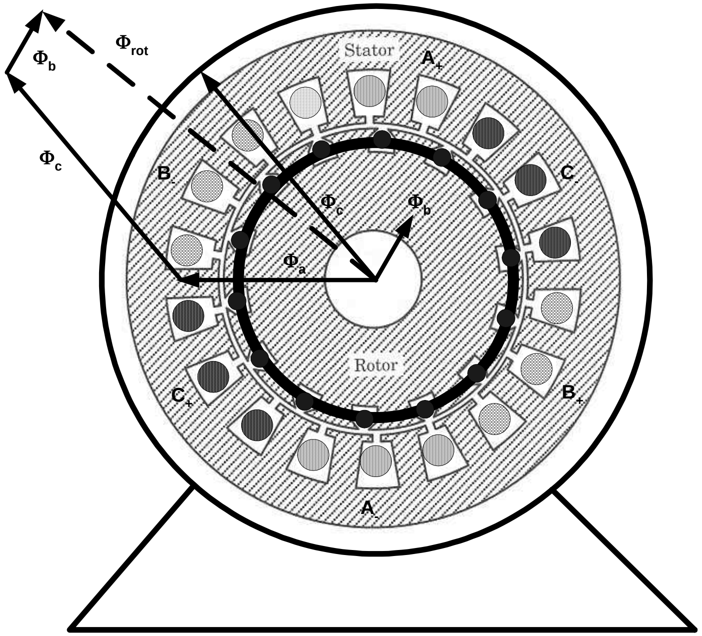
- The three-phase winding on the stator generates a rotating field.
- The rotor bar cuts the magnetic field lines as the field rotates.
- The rotating field induces a voltage in the short-circuited rotor bars
- The induced voltage is proportional to the speed difference between the rotating field and the spinning rotor



$$V = B\,L\,(v_{syn} - v_{m})$$

Induction Motors

- The speed of flux cutting is the difference between the magnetic field speed and the rotor speed.
- The two speeds can be calculated by using the radius at the rotor bar location and the rotational speed.



$$v_{syn} = 2\pi\, r_{rot}\, n_{syn}$$

$$v_{mot} = 2\pi\, r_{rot}\, n_{m}$$

$$\mathbf{V}_{\mathbf{bar}} = 2\pi\, \mathbf{r}_{\mathbf{rot}}\, \mathbf{B}\, \ell_{\mathbf{rot}} \left(\mathbf{n}_{\mathbf{syn}} - \mathbf{n}_{\mathbf{m}}\right)$$

Induction Motors

- **The voltage and current generation in the rotor bar require a speed difference between the rotating field and the rotor.**
- **Consequently, the rotor speed is always less than the magnetic field speed.**
- **The relative speed difference is the *slip*, which is calculated using**

$$s = \frac{n_{sy} - n_m}{n_{sy}} = \frac{\omega_{sy} - \omega_m}{\omega_{sy}}$$

The synchronous speed is $n_{sy} = \dfrac{f}{p/2}$

Motor Force Generation

Induction Motors

- **The interaction between the magnetic field B and the current generates a force**

$F = B\,L\,I$

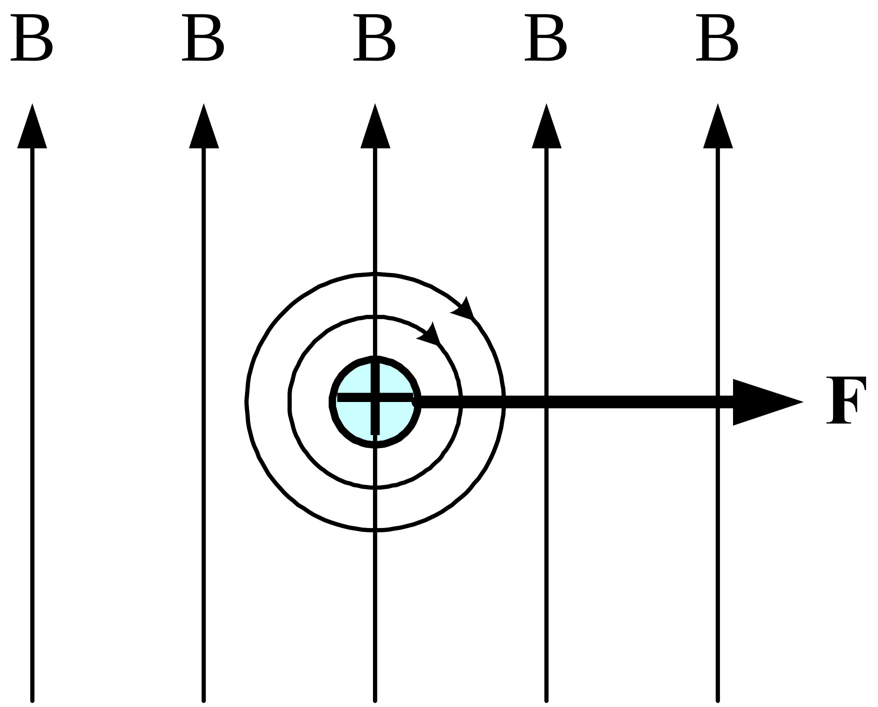


Figure 15 Force direction on a current-carrying conductor placed in a magnetic field (B) (current into the page).

Induction Motors

Force generation in a motor

- The three-phase winding generates a rotating field;
- The rotating field induces a current in the rotor bars;
- The current generation requires a speed difference between the rotor and the magnetic field;
- The interaction between the field and the current produces the driving force.

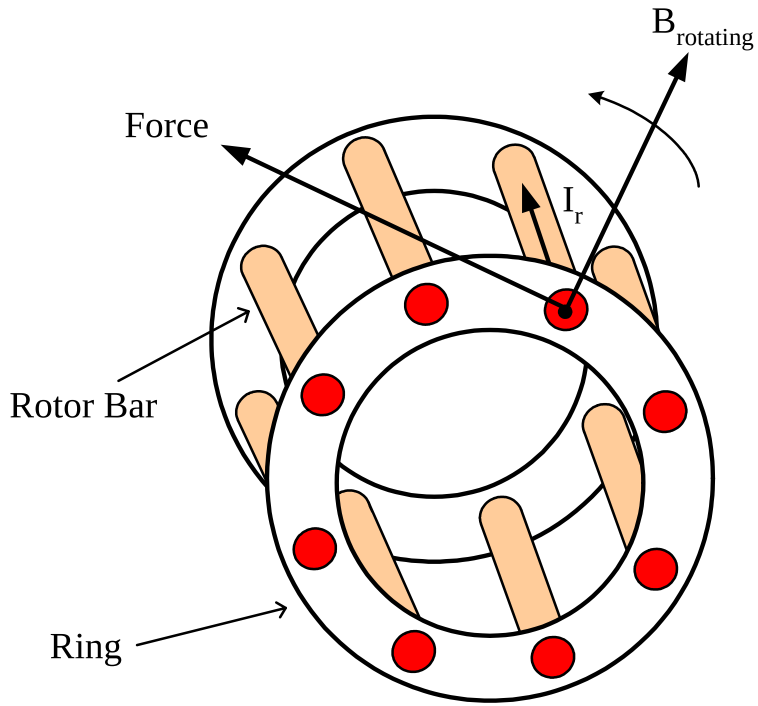


Figure 16 Rotating magnetic field generated driving force.

Equivalent circuit

Induction Motors

- An induction motor has two magnetically coupled circuits: the stator and the rotor. The latter is short-circuited.
- This is similar to a transformer, whose secondary is rotating and short-circuited.
- The motor has balanced three-phase circuits; consequently, the single-phase representation is sufficient.
- Both the stator and rotor have windings, which have resistance and leakage inductance.
- The stator and rotor winding are represented by a resistance and leakage reactance connected in series

Induction Motors

- A transformer represents the magnetic coupling between the two circuits.
- The stator produces a rotating magnetic field that induces voltage in both windings.
  - A magnetizing reactance (Xm) and a resistance connected in parallel represent the magnetic field generation.
  - The resistance (Rc) represents the eddy current and hysteresis losses in the iron core
- The induced voltage is depend on the slip and the turn ratio

Induction Motors

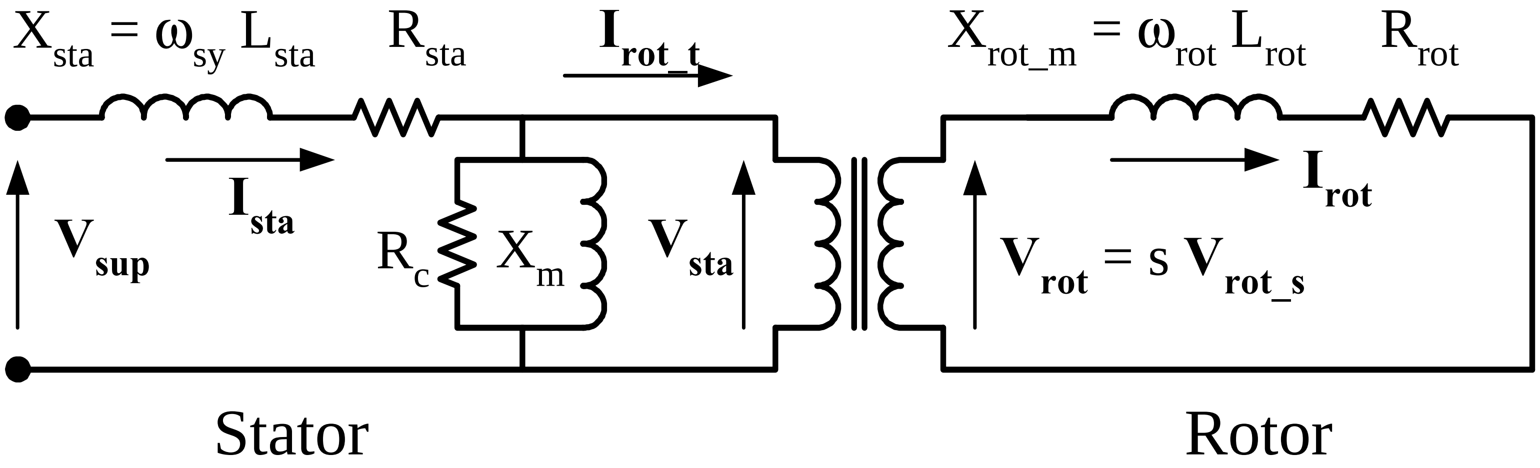


Figure 17 Single-phase equivalent circuit of a three-phase induction motor.

Induction Motors

- **In this circuit, the magnetizing reactance generates a flux that links with both the stator and the rotor and induces a voltage in both circuits.**
- **The magnetic flux rotates with constant amplitude and synchronous speed.**
- **This flux cuts the stationary conductors of the stator with the synchronous speed and induces a 60 Hz voltage in the stator windings.**
- **The rms value of the voltage induced in the stator is**:

$$V_{sta} = \frac{N_{sta}\,\Phi_{\max}\,\omega_{sy}}{\sqrt{2}}$$

Induction Motors

- The flux rotates with the synchronous speed and the rotor with the motor speed.
- Consequently, the flux cuts the rotor conductors with the speed difference between the rotating flux and the rotor.
- The speed difference is calculated using the slip equation:

$$(\omega_{sy} - \omega_m) = \omega_{sy}\ s$$

- The induced voltage is:

$$V_{rot} = \frac{N_{rot}\,\Phi_{\max}\,(\omega_{sy} - \omega_m)}{\sqrt{2}} = \frac{N_{rot}\,\Phi_{\max}\,\omega_{sy}\,s}{\sqrt{2}}$$

Induction Motors

- **The division of the rotor and stator induced voltage results in:**

$$V_{rot} = \frac{N_{rot}}{N_{sta}} V_{sta} \, s = V_{rot\_s} \, s$$

- This speed difference determines the frequency of the rotor current

$$f_{rot} = \frac{\omega_{rot}}{2\pi} = \frac{\omega_{sy} - \omega_m}{2\pi} = \frac{\omega_{sy} \, s}{2\pi} = s \; f_{sy}$$

- The rotor circuit leakage reactance is:

$$X_{rot\_m} = L_{rot} \, \omega_{rot} = L_{rot} \; \omega_{sy} \; s = X_{rot} \; s$$

Induction Motors

- **The relation between rotor current and the rotor-induced voltage is calculated by the loop voltage equation:**

$$\mathbf{V}_{\mathbf{rot}} = \mathbf{V}_{\mathbf{rot\_s}}\, s = \mathbf{I}_{\mathbf{rot}}\,(R_{rot} + j\,X_{rot}\,s)$$

- **The division of this equation with the slip yields**

$$\mathbf{V}_{\mathbf{rot\_s}} = \mathbf{I}_{\mathbf{rot}}\left(\frac{R_{rot}}{s} + j\,X_{rot}\right)$$

- The implementation of this equation simplifies the equivalent circuit

Induction Motors

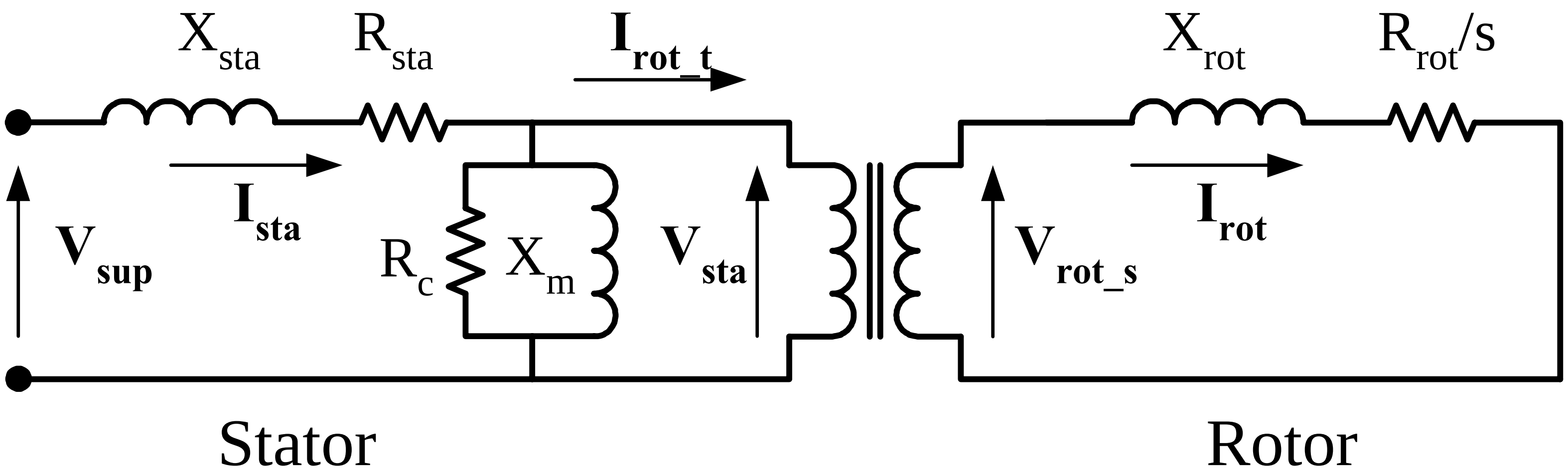


Figure 18 Modified equivalent circuit of a three-phase induction motor.

The rotor impedance is transferred to the stator side. This eliminates the transformer

Induction Motors

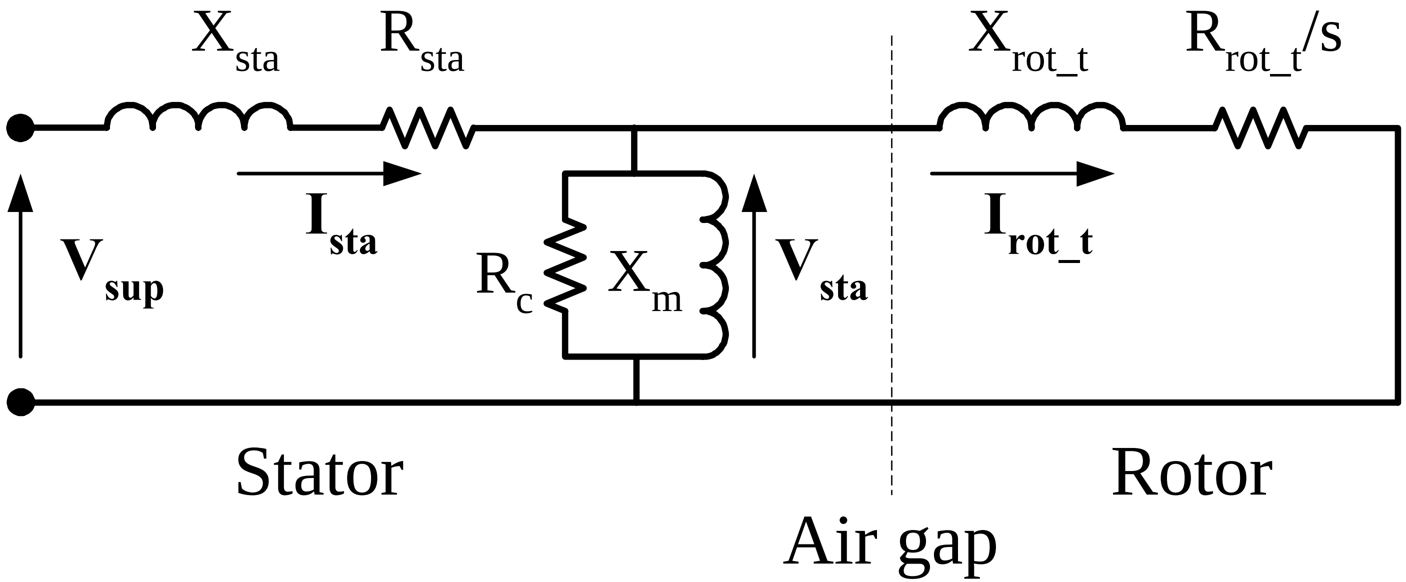


Figure 19 Simplified equivalent circuit of a three-phase induction motor.

Induction Motors

- **The last modification of the equivalent circuit is the separation of the rotor resistance into two parts:**

$$\frac{R_{rot\_t}}{s} = R_{rot\_t} + \frac{[1-s]}{s} R_{rot\_t}$$

- **The obtained resistance represents the outgoing mechanical power**

$$\frac{[1-s]}{s} R_{rot\_t}$$

Induction Motors

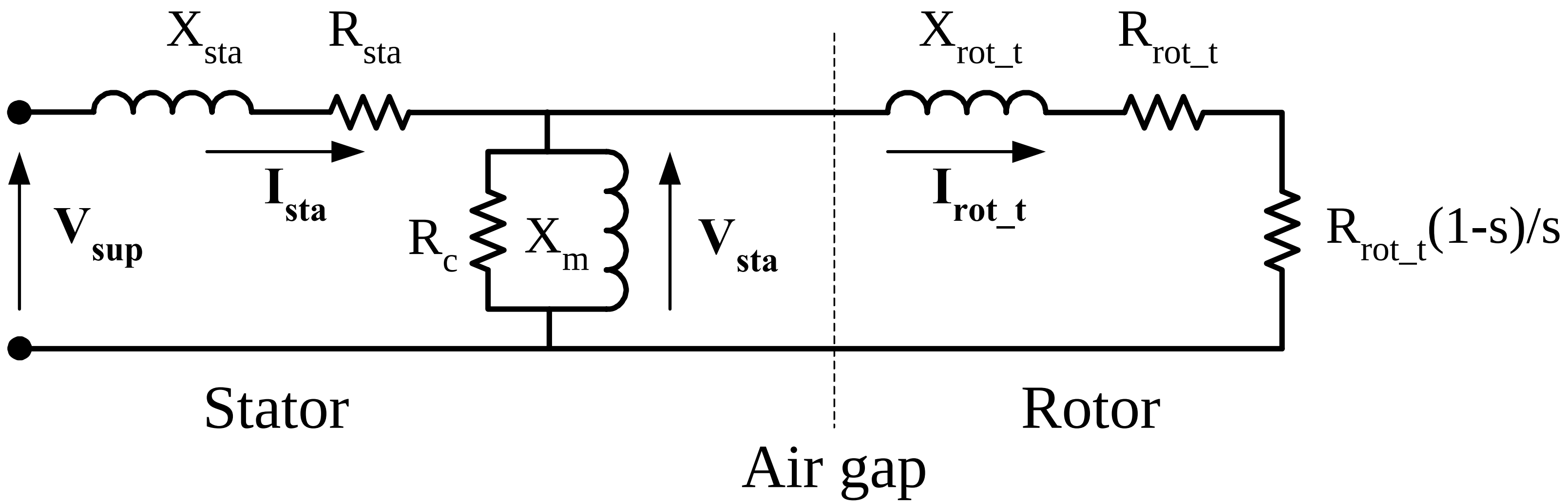


Figure 20 Final single-phase equivalent circuit of a three-phase induction motor.

Motor performance

Induction Motors

- **Figure 21 shows the energy balance in a motor.**
- **The supply power is:**

$$P_{sup} = \mathrm{Re}(\mathbf{S}_{\mathbf{sup}}) = \mathrm{Re}\left(3\,\mathbf{V}_{\mathbf{sup}}\,\mathbf{I}^{*}_{\mathbf{sta}}\right)$$

- **The power transferred through the air gap by the magnetic coupling is the input power (P_{sup}) minus the stator copper loss and the magnetizing (stator iron) loss.**
- **The electrically developed power (P_{dv}) is the difference between the air gap power (P_{ag}) and rotor copper loss.**

Induction Motors

- **The electrically developed power can be computed from the power dissipated in the second term of rotor resistance:**

$$\mathrm{P}_{\mathrm{dv}} = 3\left|\mathbf{I}_{\mathbf{rot\_t}}\right|^2 \left(R_{rot\_t} \frac{1-s}{s} \right)$$

- **The subtraction of the mechanical ventilation and friction losses (P_{mloss}) from the developed power gives the mechanical output power**

$$\mathrm{P}_{\mathrm{out}} = \mathrm{P}_{\mathrm{dv}} - \mathrm{P}_{\mathrm{mloss}}$$

Induction Motors

- **The motor efficiency:**

$$\eta = \frac{P_{out}}{P_{sup}}$$

- **Motor torque:**

$$M = \frac{P_{out}}{\omega_m}$$

Induction Motors

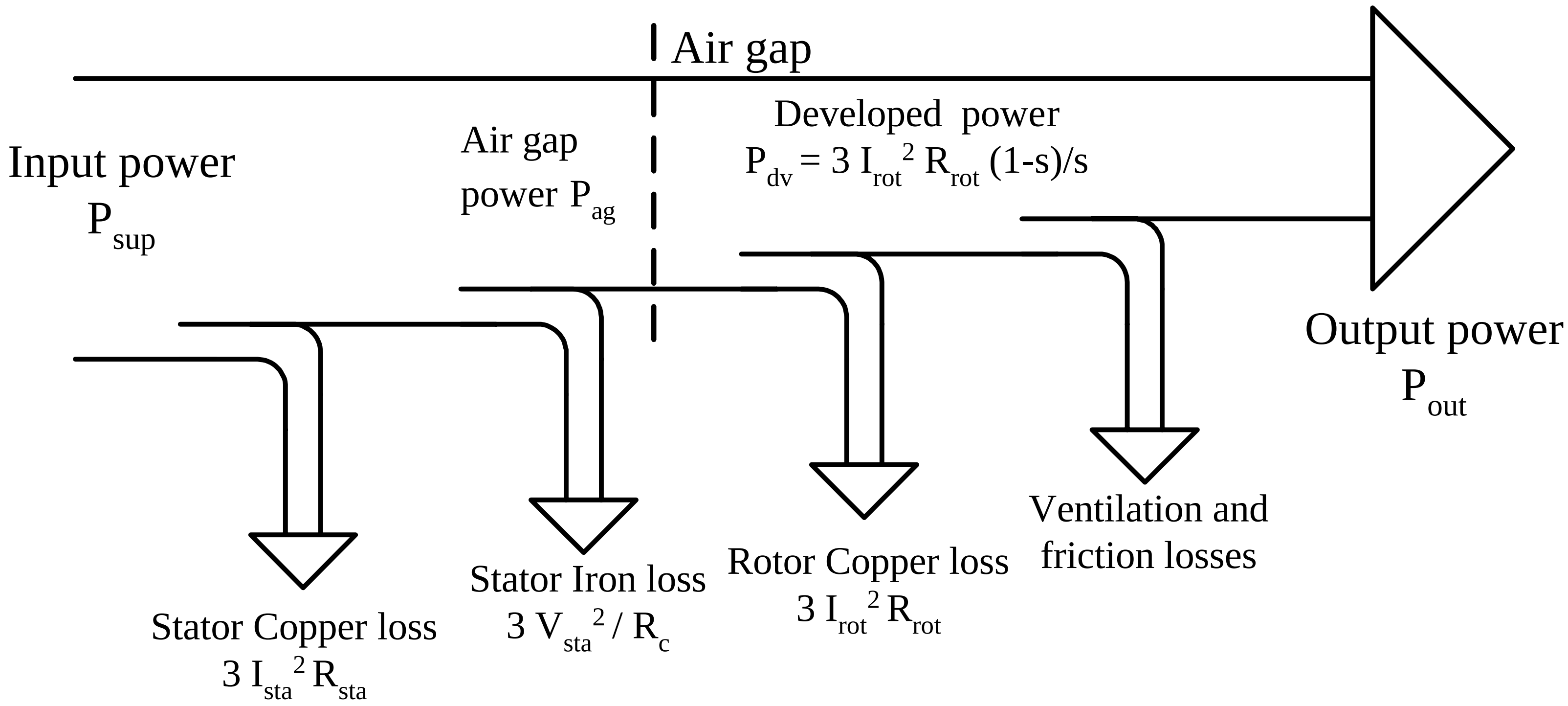


Figure 21 Motor energy balance flow diagram.

7.3.4 Motor performance analysis

Induction Motors

1) Motor impedance

$$Z_{rot\_t}(s) := j \cdot X_{rot\_t} + R_{rot\_t} + R_{rot\_t} \cdot \frac{(1 - s)}{s}$$

$$Z_m := \frac{j \cdot X_m \cdot R_c}{j \cdot X_m + R_c}$$

$$Z_{sta} := j \cdot X_{sta} + R_{sta}$$

$$Z_{mot}(s) := Z_{sta} + \frac{Z_m \cdot Z_{rot\_t}(s)}{Z_m + Z_{rot\_t}(s)}$$

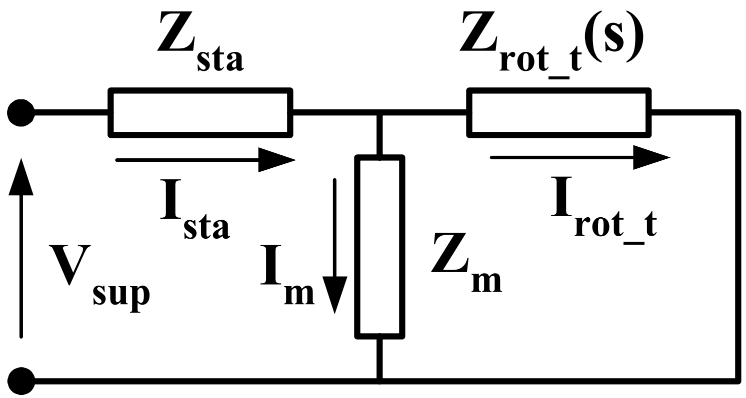


Figure 7.22 Simplified motor equivalent circuit.

Induction Motors

2) Motor Current

$$V_{sup} := \frac{V_{mot}}{\sqrt{3}}$$

$$I_{sta}(s) := \frac{V_{sup}}{Z_{mot}(s)}$$

$$I_{rot\_t}(s) := I_{sta}(s) \cdot \frac{Z_m}{Z_m + Z_{rot\_t}(s)}$$

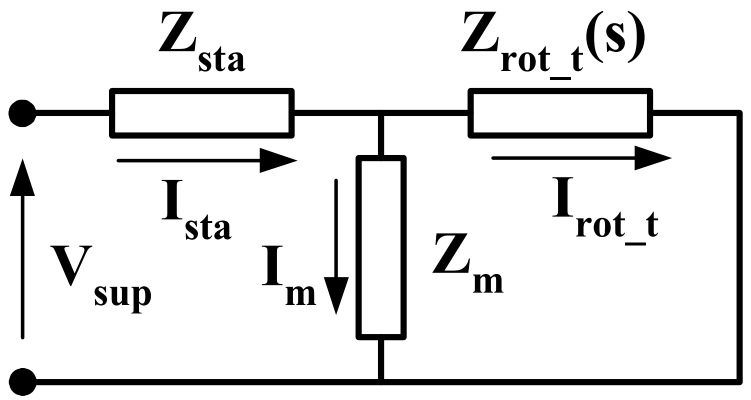


Figure 7.22 Simplified motor equivalent circuit.

Induction Motors

3) Motor Input Power

$$S_{sup}(s) := 3\cdot V_{sup}\cdot \overline{I_{sta}(s)} \qquad P_{sup}(s) := \mathrm{Re}\left(S_{sup}(s)\right)$$

$$Pf_{sup}(s) := \frac{P_{sup}(s)}{\left|S_{sup}(s)\right|} \qquad Q_{sup}(s) := \mathrm{Im}\left(S_{sup}(s)\right)$$

4) Motor Output Power and efficiency

$$P_{dev}(s) := 3\cdot\left(\left|I_{rot\_t}(s)\right|\right)^2\cdot R_{rot\_t}\cdot\frac{(1-s)}{s}$$

$$P_{mech}(s) := P_{dev}(s) - P_{mech\_loss} \qquad \eta(s) := \frac{P_{mech}(s)}{P_{sup}(s)}$$

Induction Motors

$s := 0.1 \cdot \%, 0.2\% .. 100 \cdot \%$

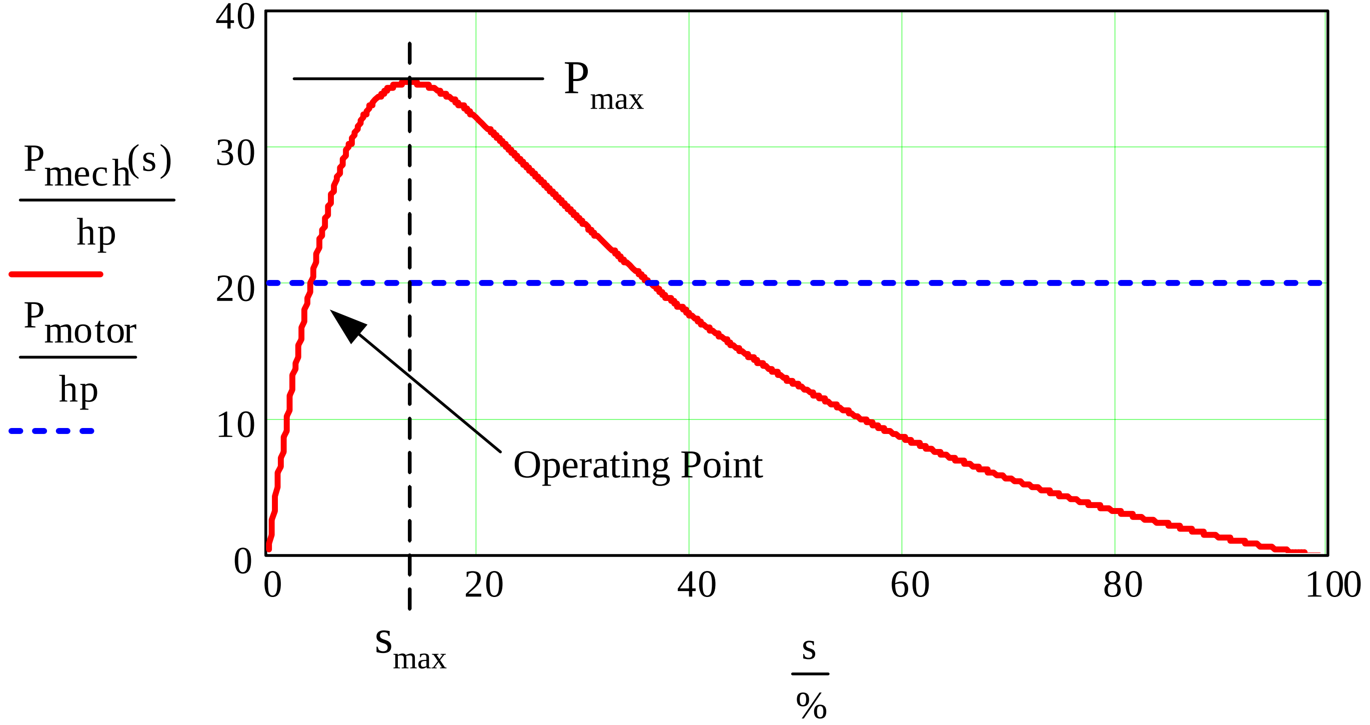


Figure 24 Mechanical output power versus slip.

Induction Motors

5. Motor Speed

$$rpm := \frac{1}{min}$$

$$n_{sy} := \frac{f}{\frac{p}{2}}$$

$$n_m(s) := n_{sy} \cdot (1 - s)$$

$$\omega_m(s) := 2 \cdot \pi \cdot n_m(s)$$

6. Motor Torque

$$T_m(s) := \frac{P_{mech}(s)}{\omega_m(s)}$$

Induction Motors

$s := 0.5\%, 0.6\% .. 80\%$

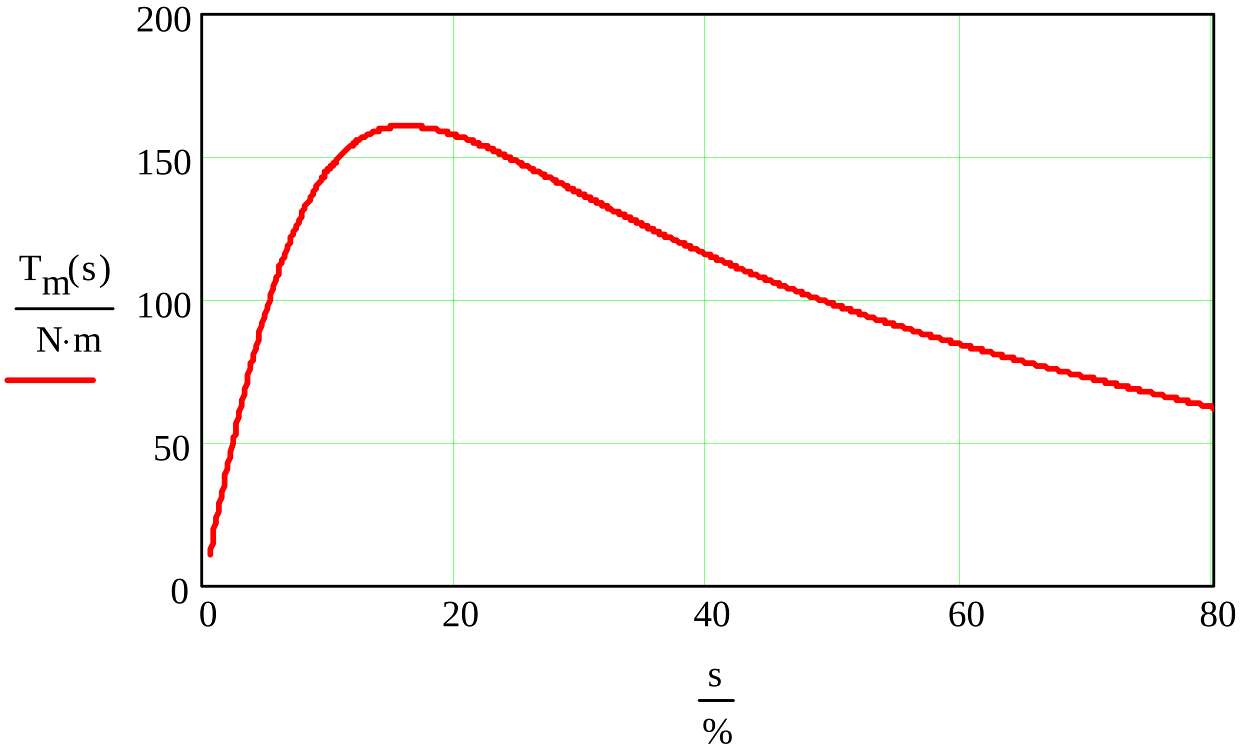


Figure 25 Torque versus slip.

Induction Motors

$s := 0.5\%, 0.6\% .. 80\%$

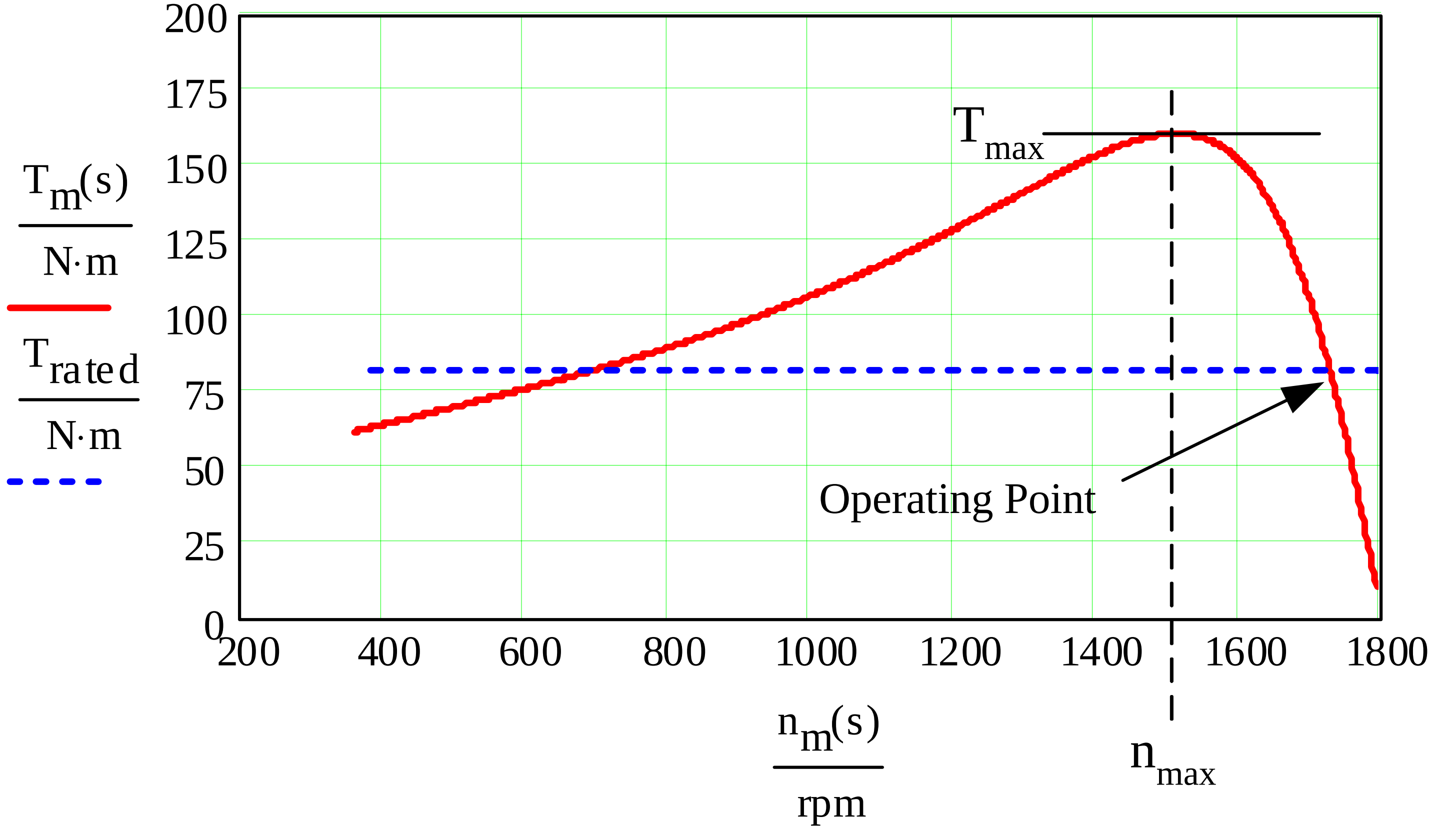
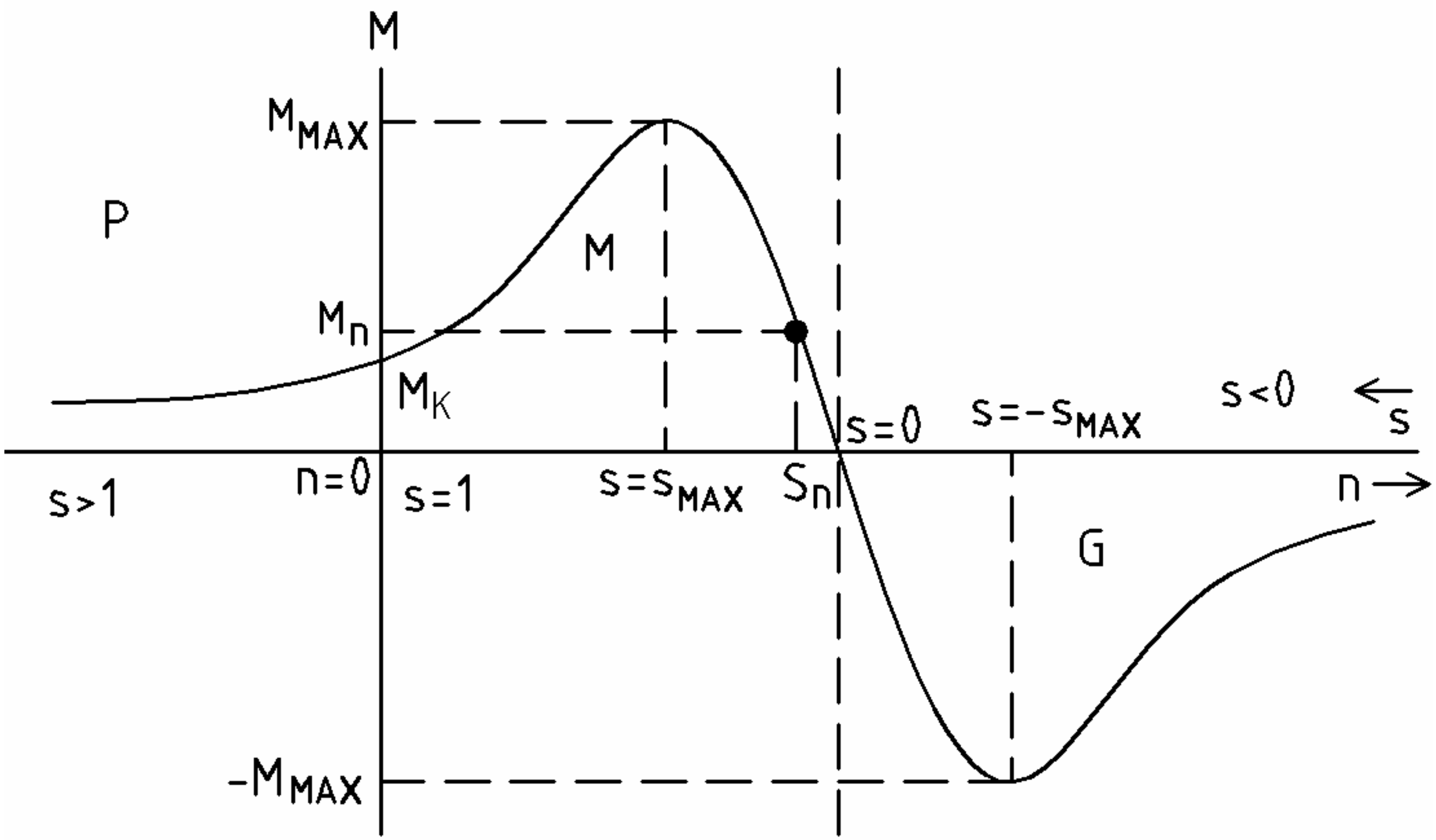


Figure 26 Torque versus speed.

Induction Motors



Induction Motors

Motor Starting torque

- **When the motor starts at $s = 1$,**
- **The ventilation losses are zero and the friction loss is passive. The negative friction loss does not drive the motor backwards.**
- **The mechanical losses are zero when $s = 1$**
- **This implies that the starting torque is calculated from the developed power instead of the mechanical output power.**

Induction Motors

Motor Starting torque

$$M_{m\_start}(s) := \frac{3\cdot\left(\left|I_{rot\_t}(s)\right|\right)^2\cdot R_{rot\_t}\cdot\frac{(1-s)}{s}}{2\cdot\pi\cdot n_{sy}\cdot(1-s)}$$

$$M_{m\_start}(s) := \frac{3\cdot\left(\left|I_{rot\_t}(s)\right|\right)^2\cdot\frac{R_{rot\_t}}{s}}{2\cdot\pi\cdot n_{sy}}$$

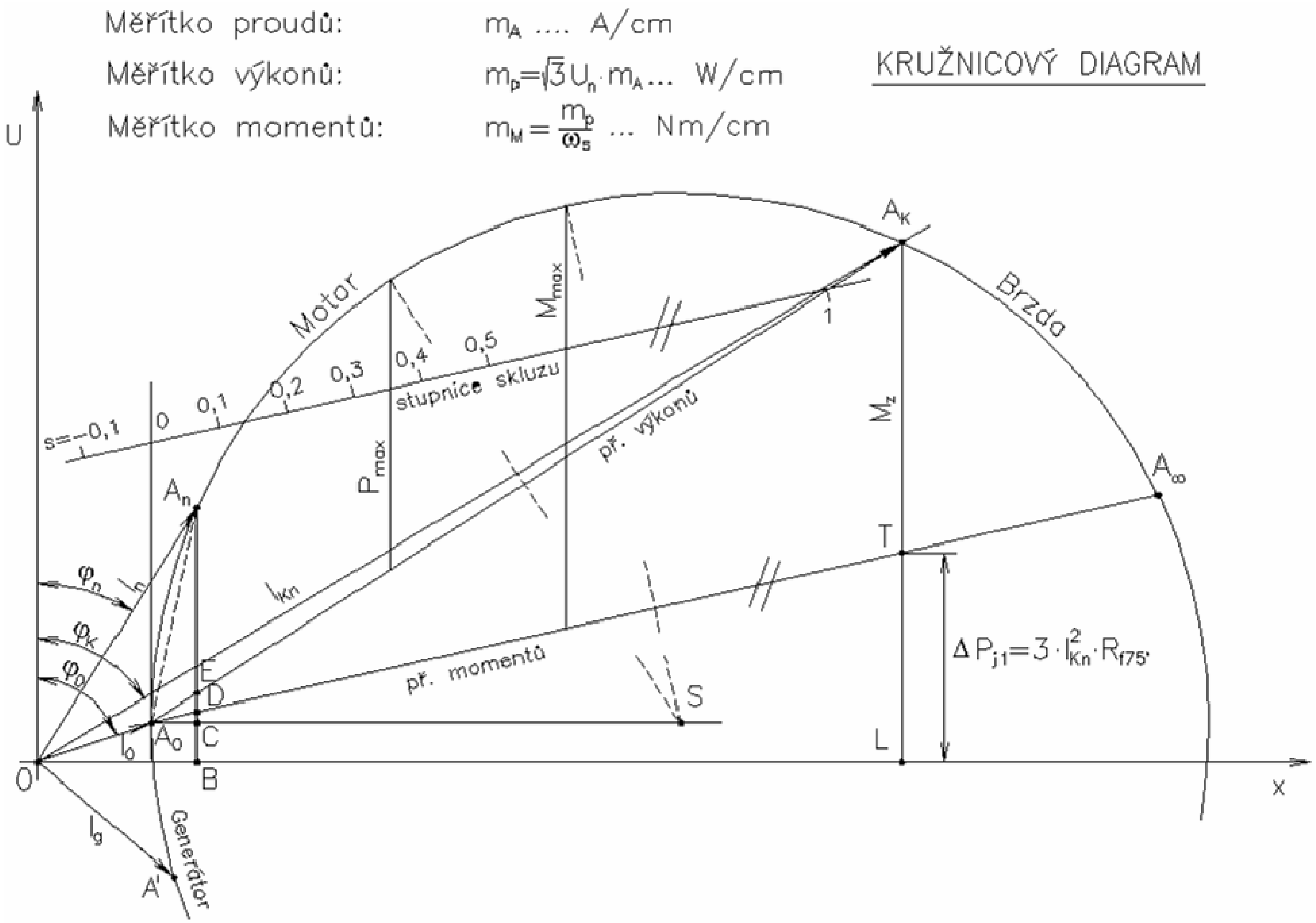
Induction Motors

- **Kloss formula**

$$M = \frac{2 M_{MAX}}{\frac{s_{MAX}}{s} + \frac{s}{s_{MAX}}}$$

Induction Motors

Circular diagram



Induction Motors

- The resistances and reactance in the equivalent circuit for an induction motor can be determined by a series of measurements. The measurements are:
  - <u>No-load test</u>. This test determines the magnetizing reactance and core loss resistance.
  - <u>Blocked-rotor test</u>. This test gives the combined value of the stator and rotor resistance and reactance.
  - <u>Stator resistance measurement</u>.

Induction Motors

No-load test

- The motor shaft is free
- The rated voltage supplies the motor.
- In the case of a three-phase motor:
  - the line-to-line voltages,
  - line currents
  - three-phase power using two wattmeters are measured

Induction Motors

No-load test

Fig 29 Equivalent motor circuit in no-load test

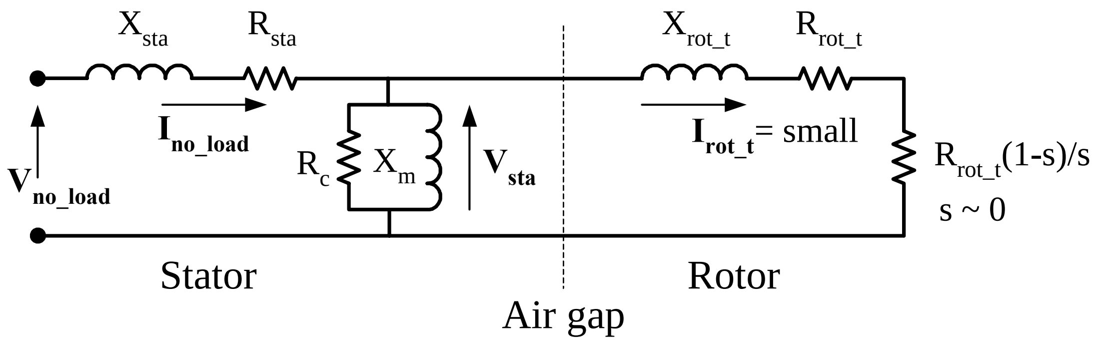
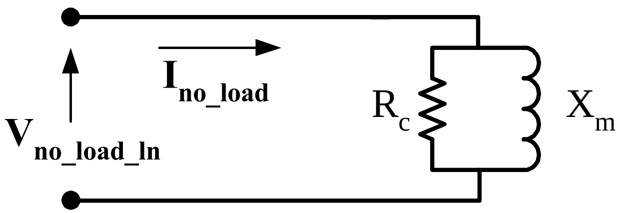


Fig. 30 Simplified equivalent motor circuit in no-load test



Induction Motors

No-load test

$$V_{no\_load\_ln} := \frac{V_{no\_load}}{\sqrt{3}}$$

$$R_c := \frac{V_{no\_load\_ln}^2}{P_{no\_load\_A}}$$

$$P_{no\_load\_A} := \frac{P_{no\_load}}{3}$$

$$S_{no\_load\_A} := V_{no\_load\_ln} \cdot I_{no\_load}$$

$$Q_{no\_load\_A} := \sqrt{S_{no\_load\_A}^2 - P_{no\_load\_A}^2}$$

$$X_m := \frac{V_{no\_load\_ln}^2}{Q_{no\_load\_A}}$$

Induction Motors

Blocked-rotor test

- The rotor is blocked to prevent rotation
- The supply voltage is reduced until the motor current is around the rated value.
- The motor is supplied by reduced voltage and reduced frequency. The supply frequency is typically 15 Hz.
- In the case of a three-phase motor:
  - the line-to-line voltages,
  - line currents
  - three-phase power using two wattmeters are measured

Induction Motors

Blocked-Rotor test

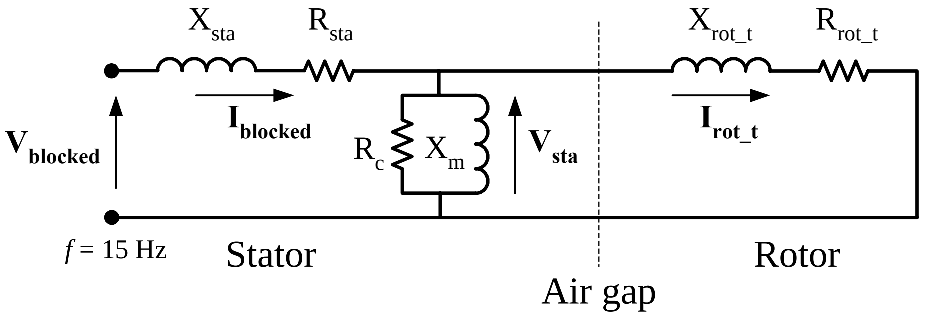


Figure 31 Equivalent motor circuit for blocked-rotor test

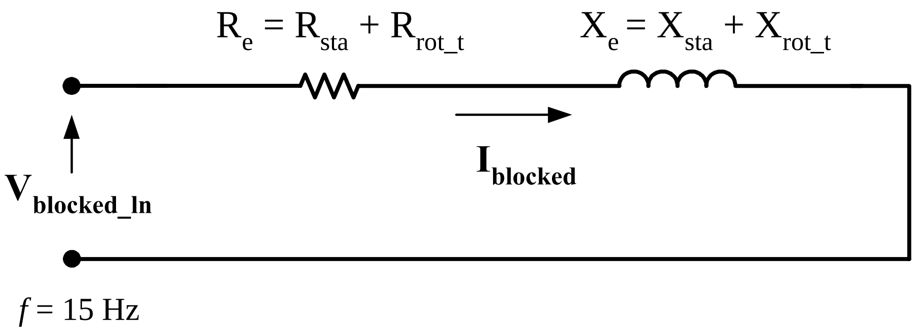


Figure 32 Simplified equivalent motor circuit for blocked-rotor test

Induction Motors

Blocked-Rotor test

$$V_{blocked\_ln} := \frac{V_{blocked}}{\sqrt{3}}$$

$$R_e := \frac{P_{blocked\_A}}{I_{blocked}^{\ 2}}$$

$$P_{blocked\_A} := \frac{P_{blocked}}{3}$$

The stator resistance was measured directly

$$R_{rot\_t} := R_e - R_{sta}$$

Induction Motors

Blocked-Rotor test

The magnitude of the motor impedance

$$Z_{blocked} := \frac{V_{blocked\_ln}}{I_{blocked}}$$

The leakage reactance at 15 Hz

$$X_{e\_15Hz} := \sqrt{Z_{blocked}^{\;2} - R_e^{\;2}}$$

The leakage reactance at 60 Hz

$$X_e := X_{e\_15Hz} \cdot \frac{60Hz}{15Hz}$$

Numerical Example

Induction Motors

A three -phase 30hp, 208V, 4 pole, 60Hz, wye connected induction motor was tested, the obtained results are:

No load test, 60 Hz $V_{nL} := 208V$ $P_{nL} := 1600W$ $I_{nL} := 22A$

Blocked Rotor test, 15Hz $V_{br} := 21V$ $P_{br} := 2100W$ $I_{br} := 71A$ $f_{br} := 15Hz$

DC test $V_{dc} := 12V$ $I_{dc} := 75A$

Motor rating: $P_{mot\_rated} := 30hp$ $V_{mot\_ll} := 208V$ $pf_{mot} := 0.8$

$p := 4$ $f := 60Hz$

Calculate :

a) The equivalent circuit parameters

b) Motor rated current and synchronous speed

Draw the equivalent circuit

Induction Motors

No load test , Determine the core losses Rc and magnetizing reactance Xm

Single phase values:

$$P_{nL\_1F} := \frac{P_{nL}}{3} \qquad V_{nL\_ln} := \frac{V_{nL}}{\sqrt{3}}$$

$$R_c := \frac{\left(V_{nL\_ln}\right)^2}{P_{nL\_1F}} \qquad R_c = 27.04\Omega$$

$$Y_{nL} := \frac{I_{nL}}{V_{nL\_ln}} \qquad Y_{nL} = 0.183S$$

$$X_m := i \cdot \frac{1}{\sqrt{Y_{nL}^{\,2} - \left(\frac{1}{R_c}\right)^2}} \qquad X_m = 5.573i\Omega$$

Induction Motors

Block Rotor test,

Neglect the magnetizing branch. Consider only the $X_{sta}+X_{rot}$ and $R_{sta}+R_{rot}$

$X_{br} = X_{sta} + X_{rot}$ $\qquad$ $R_{br} = R_{sta} + R_{rot}$

Single phase values: $\quad P_{br\_1F} := \frac{P_{br}}{3} \quad V_{br\_ln} := \frac{V_{br}}{\sqrt{3}}$

Resistance value is:

$R_{br} := \frac{P_{br\_1F}}{I_{br}^{2}} \qquad R_{br} = 0.139\Omega$

Induction Motors

$$Z_{br} := \frac{V_{br\_ln}}{I_{br}} \qquad Z_{br} = 0.171\Omega$$

$$X_{br\_15Hz} := i \cdot \sqrt{Z_{br}^{2} - R_{br}^{2}} \qquad X_{br\_15Hz} = 0.099i\Omega$$

The reactance at 60 Hz is:

$$X_{br\_60Hz} := X_{br\_15Hz} \cdot \frac{60}{15} \qquad X_{br\_60Hz} = 0.398i\Omega$$

$$X_{br} := i \cdot \sqrt{\left(Z_{br}^{2} - R_{br}^{2}\right)} \cdot \frac{60Hz}{f_{br}} \qquad X_{br} = 0.398i\Omega$$

Induction Motors

Determination of R1 and R2 and X1 and X2

$X_{sta} := \frac{X_{br}}{2}$ $X_{rot} := X_{sta}$ $X_{sta} = 0.199i\Omega$

Y connected motor

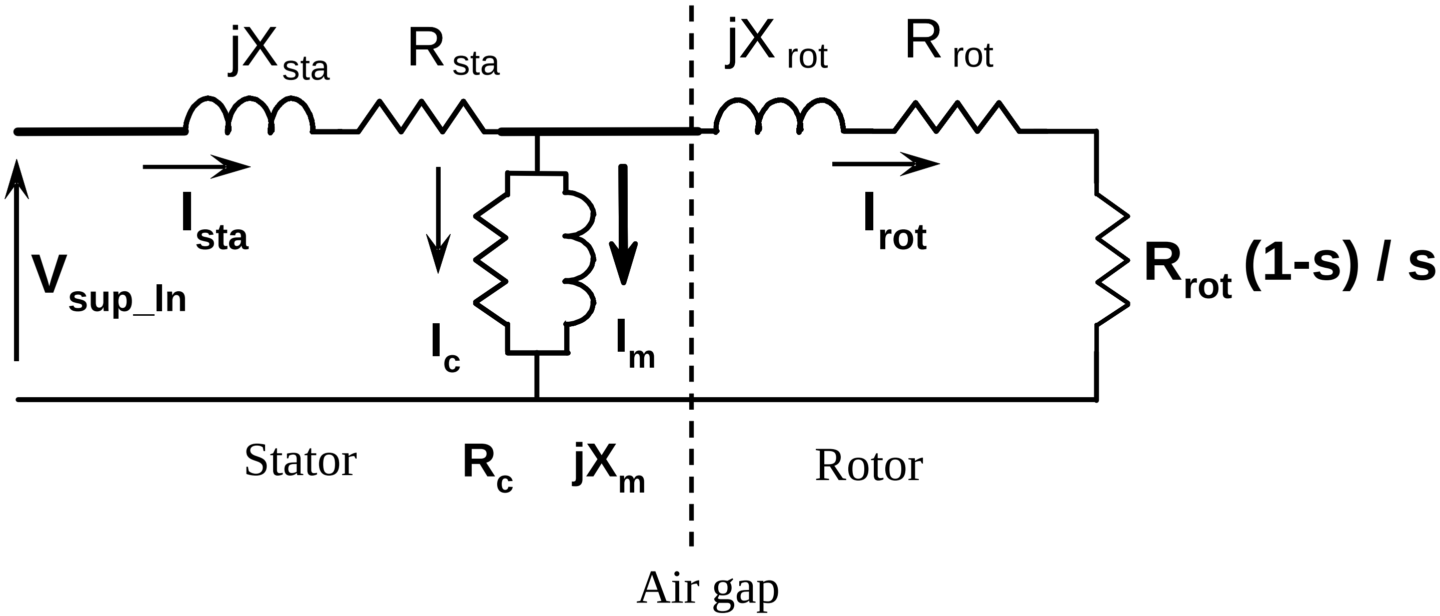
$R_{sta} := \frac{V_{dc}}{2I_{dc}}$ $R_{rot} := R_{br} - R_{sta}$

$R_{sta} = 0.08\Omega$ $R_{rot} = 0.059\Omega$

Induction Motors

a) The equivalent circuit parameters

a) The equivalent circuit parameters



Induction Motors

C) Motor rated current and synchronous speed

$$S_{rated} := \frac{P_{mot\_rated}}{pf_{mot}}$$

$$S_{rated} = 27.964 kV \cdot A$$

$$I_{mot\_rated} := \frac{S_{rated}}{\sqrt{3} \cdot V_{mot\_ll}}$$

$$I_{mot\_rated} = 77.62 A$$

$$rpm := \frac{1}{min}$$

$$n_{synch} := \frac{f}{\frac{p}{2}}$$

$$n_{synch} = 30 Hz$$

$$n_{synch} = 1800 rpm$$