Transformers
Operating principle of a transformer

- Transformers are stationary electrical machines which transmit energy from systems with certain current and voltage values into systems with generally different current and voltage values but with identical frequency.
Operating principle of a transformer

• Two separate windings are on the same iron core. Following connection to alternating voltage $U_1$ there is a standstill current $I$. The magnetomotive force $H = I_1 \cdot N_1$ generates a magnetic alternating flow ($\Phi_1$) in the iron core.

• The input and output winding of an alternating voltage are induced in accordance with the induction law. A self-induction voltage $U_{10}$ arises in the input winding. It is counter-positioned in accordance with Lenz's law on applied voltage. During idling operation - because of mutual induction - there arises the output voltage $U_{20}$ which is simultaneously the terminal voltage $U_2$. 
Operating principle of a transformer

The value of the induced voltage is derived from the following equation:

\[ U_0 = 4,44 \times N \times B \times A_{Fe} \times f \]

where:
- \( U_0 \) induction voltage
- \( N \) number of turns
- \( B \) max. flow density
- \( A_{Fe} \) limb cros-section
- \( f \) induction voltage frequency

The induction voltage increases along with the number of turns, the magnetic flow density in the iron core, the iron cross-section and the frequency.
Operating principle of a transformer

Example:
Which maximum flow density occurs in an iron core of 16 cm² cross-section when a voltage of 380 V (50 Hz) is applied to the primary coil with 930 turns?

Given: $A_{Fe} = 16 \text{ cm}^2$; $N_1 = 930$; $U_1 = 380 \text{ V}$; $f = 50 \text{ Hz}$

Solution: 1.15 T
Voltage transformation

A few field lines already close before reaching the output coil so that flow $\Phi_1$ can be divided into a maximum flow $\Phi_K$ which saturates both coils and a leakage flow $\Phi_S$.
Voltage transformation

\[ U_{10} = 4.44 \cdot N_1 \cdot \Phi_K \cdot f \]

\[ U_{20} = 4.44 \cdot N_2 \cdot \Phi_K \cdot f \]

Shortening (neglet leakage fluxes) gives us transformer ratio \( p \):

\[ p = \frac{U_1}{U_2} = \frac{N_1}{N_2} \]
Voltage transformation

The rated voltages \( U_{1n} \) and \( U_{2n} \) are indicated on the rating plate of the transformer.

Example:
What secondary terminal voltage arises in a transformer where 380 V is applied to the primary winding of 980 turns and the secondary winding has 594 turns?

Given: \( U_1 = 380 \text{ V}; N_1 = 980; N_2 = 594 \)

Sought: \( U_2 \)

Solution: \( U_2 = 230 \text{ V} \)
Load behaviour of the transformer

If the transformer is output-loaded, current $I_2$ flows into coil $N_2$. Current $I_2$ generates the magnetic flow $\Phi_{2K}$. According to Lenz's Law this magnetic flow is counter-positioned to the cause ($\Phi_{1K}$).
Load behaviour of the transformer

In this manner the magnet flow $\Phi_{1K}$ is weakened and induction voltage $U_{10}$ decreases. Given uniform rated voltage, the difference increases between the two voltages $U_{10}$ and $U_1$.

Consequently, a greater input current $I_1$ flows whereby the magnetic flow $\Phi_{1K}$ is increased. The magnetic flow $\Phi$ in the iron core thus remains virtually constant:

$$\Phi_{1K} = \Phi_{1K} - \Phi_{2K} = \text{constant}$$

This also applies to the output voltage of the transformer.

The input current $I_1$ increases as the load current $I_2$ becomes greater.
Load behaviour of the transformer

Transformation ratio
Without heeded the losses of the transformer, the following applies according to the energy conservation law:

\[ S_1 = S_2 \quad \text{and} \quad U_1 \cdot I_1 = U_2 \cdot I_2 \]

If we arrange the equation so that the voltage and current values appears on respective sides, then

\[ \frac{I_1}{I_2} = \frac{U_2}{U_1} = \frac{N_2}{N_1} = \frac{1}{p} \]
Load behaviour of the transformer

Currents the are conversely proportional to the voltages or numbers of turns. A transformer converts high currents into low ones or low currents into higher ones.

Example:
A welding transformer takes up 220 (current being 10A). The output voltage is 20 V. How great is the welding current?
Solution:
$I_2 = 110$ V
Idling behaviour

A transformer idles where mains voltage $U_1$ remains applied to the primary side whilst no consumer is connected to the secondary side ($Z_a = \infty$).

Primary circuit $U_1$ applies $I_0$ flows (idling current)
Secondary circuit $Z_a = \infty$, $I_2 = 0$, $U_2 = U_{20}$
Idling behaviour

Idling current $I_0$

The applied voltage $U_{10}$ drives the idling current $I_0$. This is needed to establish the magnetic field $\Phi$. This lags behind the voltage $U_1$. 
Idling behaviour

The value of idling current $I_0$ is between 2 and 5 % of rated current in big transformers and up to 15 % in smaller transformers.
No-load curve

The idling curve $I_0 = f(U_{10})$ in Figure indicates that no-load current $I_0$ increases proportionally to the input voltage $U_1$. No-load current increases markedly over and beyond the input rated speed $U_{1n}$. It can, moreover, even attain values greater than the rated current.

Transformers shall not be driven by voltages greater than the rated voltage $U_{1n}$.
Short-circuit behaviour

- **Short-circuit curves**
- Secondary current $I_2$ increases if load resistance is decreased. Where $Z_a = 0$ the transformer has been short-circuited.
- Primary circuit $U_1$ is applied
  $I_K$ flows
  Secondary circuit $Z_a = 0$
  $U_2 = 0$ Short-circuit voltage
- The short-circuited transformer can be replaced by resistor $Z_1$ which corresponds to the transformer internal resistor
Short-circuit behaviour

The relative short-circuit voltage $u_K$ in % is determined by the following equation:

$$u_K = \frac{U_K}{U_{1n}} \times 100\%$$

The relative short-circuit voltage is, on average, 2 to 10% of input rated voltage ($U_{1n}$) in mains transformers.
Short-circuit behaviour

Short-circuit losses (winding losses)

• In the short-circuit experiment a power meter indicates short-circuit losses as the primary and secondary rated currents generate winding losses. The iron core is only slightly magnetised by the applied short-circuit voltage

\[ U_k \ll U_1 \]

• The winding losses can be metered during the short-circuit experiment. They are dependent on the load current \( P_{ww} = I^2R \).
Loaded voltage behaviour

• In contrast to operational idling, during loading the secondary circuit is closed through an external resistance $Z_a$. Secondary current $I_2$ flows. According to the energy conservation law the transformer must also take up commensurate primary power, thus a primary current $I_1$ also flows.

• Primary circuit $U_1$ is applied
  $I_1 > I_0$  Secondary circuit $Z_a < \infty$
  $I_2 > 0$
  $U_2 < U_{20}$
Loaded voltage behaviour

- Voltage curve $U_2 = f(I_2)$
- As the curve in figure shows, terminal voltage $U_2$ decreases during loading.

1 $u_K$ small, 2 $u_K$ big
**Loaded voltage behaviour**

Secondary terminal voltage depending on the degree and nature of loading

1. Idling
2. Rated load

The output voltage of a transformer depends on the
- degree of load current $I_2$
- the magnitude of relative short-circuit voltage
- the nature of the load (ohmic, inductive or capacitive).
Full equivalent circuit diagram

\[ U'_2 = p \cdot U_2 \]
\[ L'_2 = p^2 \cdot L_2 \]
\[ I'_2 = \frac{1}{p} \cdot I_2 \]
\[ R'_2 = p^2 \cdot R_2 \]