# Transformers

# Operating principle of a transformer

- Transformers are stationary electrical machines which transmit energy from systems with certain current and voltage values into systems with generally different current and voltage values but with identical frequency

# Operating principle of a transformer

- Two separate windings are on the same iron core. Following connection to alternating voltage U1 there is a standstill current I. The magnetomotive force $H = I_1 \cdot N_1$ generates a magnetic alternating flow ($\Phi_1$) in the iron core.

- The input and output winding of an alternating voltage are induced in accordance with the induction law. A self-induction voltage $U_{10}$ arises in the input winding. It is counter-positioned in accordance with Lenz's law on applied voltage. During idling operation - because of mutual induction - there arises the output voltage $U_{20}$ which is simultaneously the terminal voltage $U_2$.

# Operating principle of a transformer

The value of the induced voltage is derived from the following equation:

$$\boxed{U_0 = 4{,}44 * N * B * A_{Fe} * f}$$

where:

| | |
|---|---|
| $U_0$ | induction voltage |
| $N$ | number of turns |
| $B$ | max. flow density |
| $A_{Fe}$ | limb cros-section |
| $f$ | induction voltage frequency |

The induction voltage increases along with the number of turns, the magnetic flow density in the iron core, the iron cross-section and the frequency.

# Operating principle of a transformer

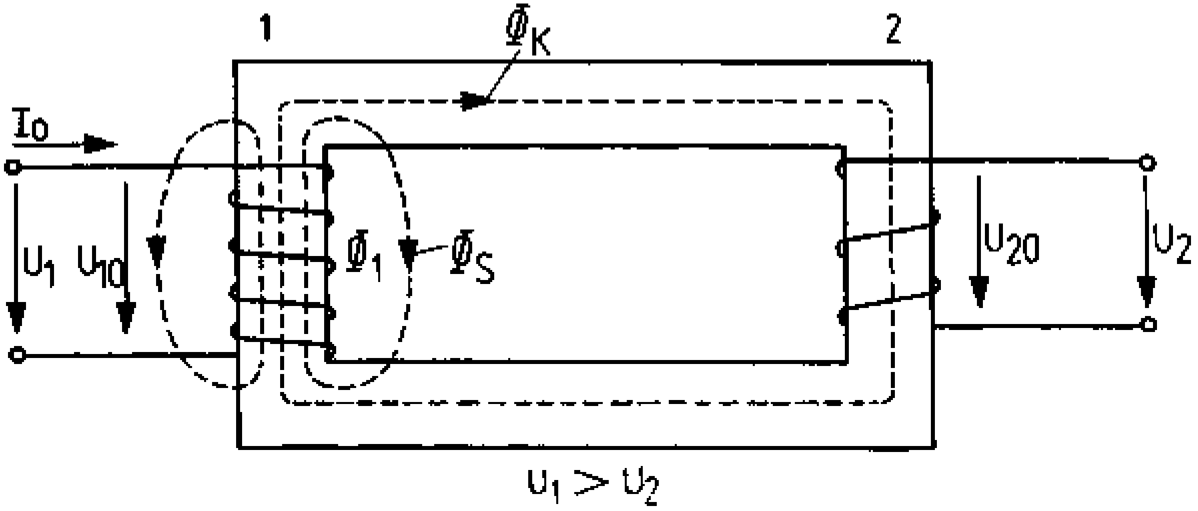
## Example:

Which maximum flow density occurs in an iron core of 16 cm$^2$ cross-section when a voltage of 380 V (50 Hz) is applied to the primary coil with 930 turns?

Given: $A_{Fe}$ = 16 cm$^2$; $N_1$ = 930; $U_1$ = 380 V; f = 50 Hz

Solution: 1.15 T

# Voltage transformation



A few field lines already close before reaching the output coil so that flow $\Phi_1$ can be divided into a maximum flow $\Phi_K$ which saturates both coils and a leakage flow $\Phi_S$

# Voltage transformation

$U_{10} = 4.44 \cdot N_1 \cdot \Phi_K \cdot f$

$U_{20} = 4.44 \cdot N_2 \cdot \Phi_K \cdot f$

Shortening (neglet leakage fluxes) gives us transformer ratio p:

$$\boxed{p = \frac{U_1}{U_2} = \frac{N_1}{N_2}}$$

# Voltage transformation

The rated voltages

**$U_{1n}$ and $U_{2n}$**

are indicated on the rating plate of the transformer

**Example:**

What secondary terminal voltage arises in a transformer where 380 V is applied to the primary winding of 980 turns and the secondary winding has 594 turns?

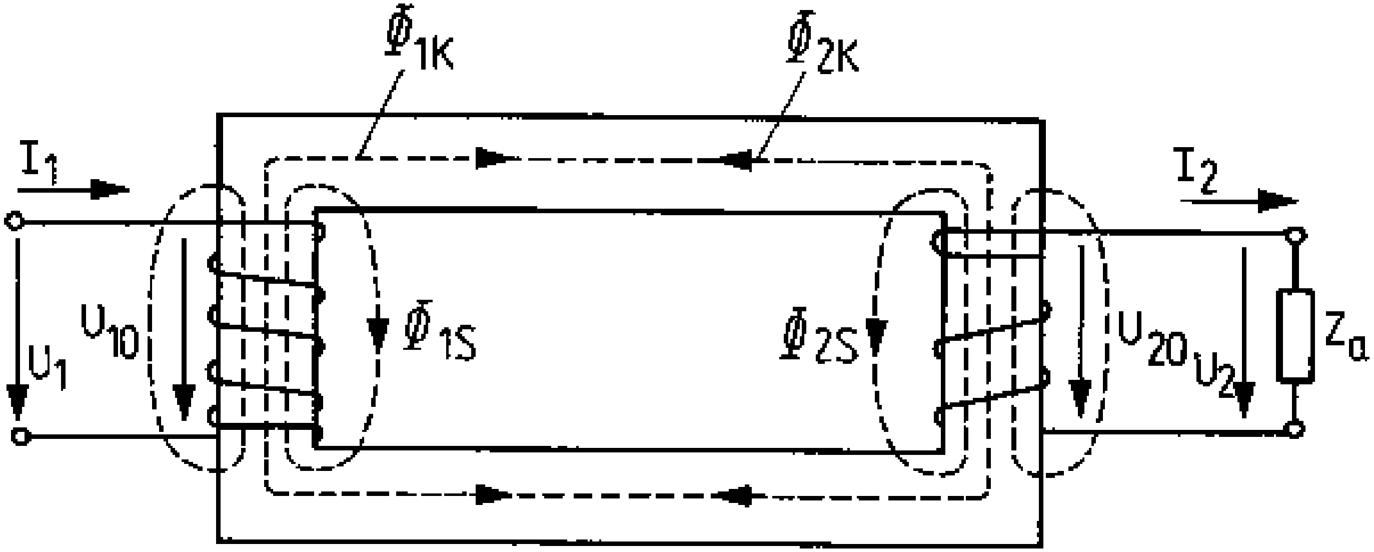
Given: $U_1 = 380$ V; $N_1 = 980$; $N_2 = 594$

Sought: $U_2$

Solution: $U_2 = 230$ V

# Load behaviour of the transformer

If the transformer is output-loaded, current $I_2$ flows into coil $N_2$. Current $I_2$ generates the magnetic flow $\Phi_{2K}$. According to Lenz's Law this magnetic flow is counter-positioned to the cause ($\Phi_{1K}$).



# Load behaviour of the transformer

In this manner the magnet flow $\Phi_{1K}$ is weakened and induction voltage $U_{10}$ decreases. Given uniform rated voltage, the difference increases between the two voltages $U_{10}$ and $U_1$.

Consequently, a greater input current $I_1$ flows whereby the magnetic flow $\Phi_{1K}$ is increased. The magnetic flow $\Phi$ in the iron core thus remains virtually constant:

$$\Phi_{1K} = \Phi_{1K} - \Phi_{2K} = \text{constant}$$

This also applies to the output voltage of the transformer.

The input current $I_1$ increases as the load current $I_2$ becomes greater.

# Load behaviour of the transformer

## Transformation ratio

Without heeded the losses of the transformer, the following applies according to the energy conservation law:

$$S_1 = S_2 \quad \text{and} \quad U_1 \cdot I_1 = U_2 \cdot I_2$$

If we arrange the equation so that the voltage and current values appears on respective sides, then

$$\frac{I_1}{I_2} = \frac{U_2}{U_1} = \frac{N_2}{N_1} = \frac{1}{p}$$

# Load behaviour of the transformer

Currents the are conversely proportional to the voltages or numbers of turns. A transformer converts high currents into low ones or low currents into higher ones.

**Example:**

A welding transformer takes up 220 (current being 10A). The output voltage is 20 V. How great is the welding current?

Solution:

$I_2$ = 110 V

# Idling behaviour

A transformer idles where mains voltage $U_1$ remains applied to the primary side whilst no consumer is connected to the secondary side
($Z_a = \infty$).

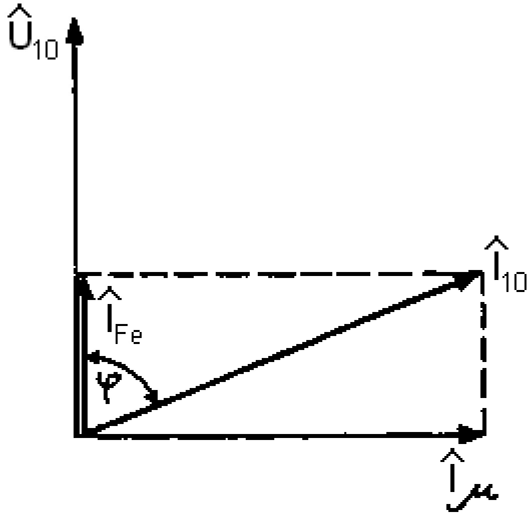
Primary circuit $U_1$ appliesI$_0$ flows (idling current)

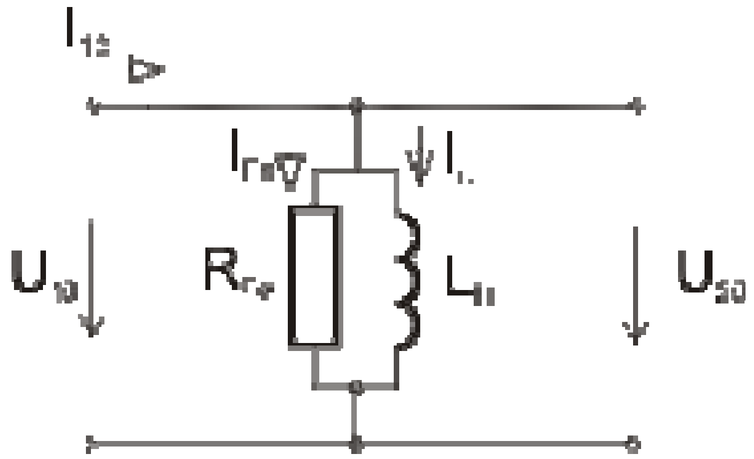
Secondary circuit $Z_a = \infty$, $I_2 = 0$, $U_2 = U_{20}$

# Idling behaviour

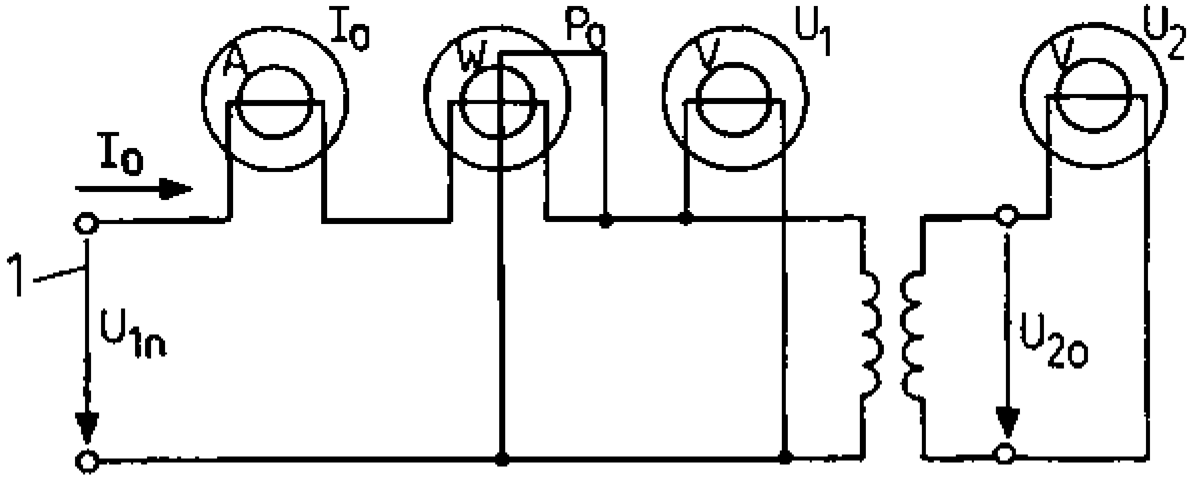
Idling current $I_0$

The applied voltage $U_{10}$ drives the idling current $I_0$. This is needed to establish the magnetic field $\Phi$.This lags behind the voltage $U_1$.





# Idling behaviour



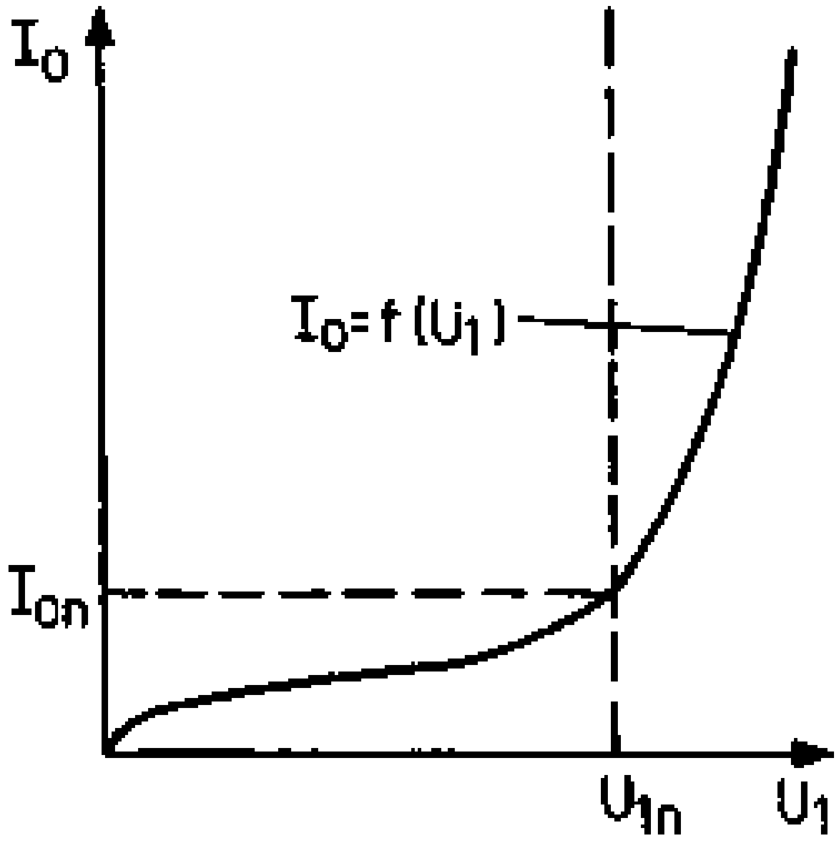
$U_{1n}$ … rated voltage

The value of idling current $I_0$ is between 2 and 5 % of rated current in big transformers and up to 15 % in smaller transformers

# No-load curve

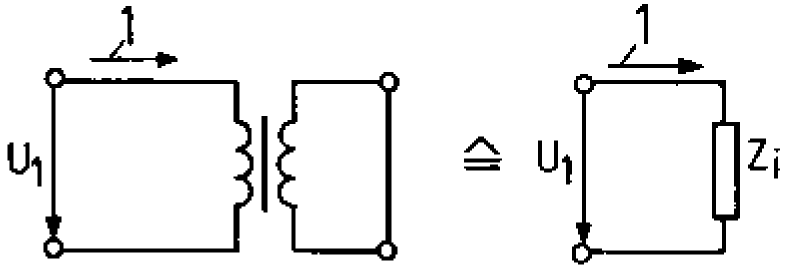
The idling curve $I_0 = f(U_{10})$ in Figure indicates that no-load current $I_0$ increases proportionally to the input voltage $U_1$. No-load current increases markedly over and beyond the input rated speed $U_{1n}$. It can, moreover, even attain values greater than the rated current.

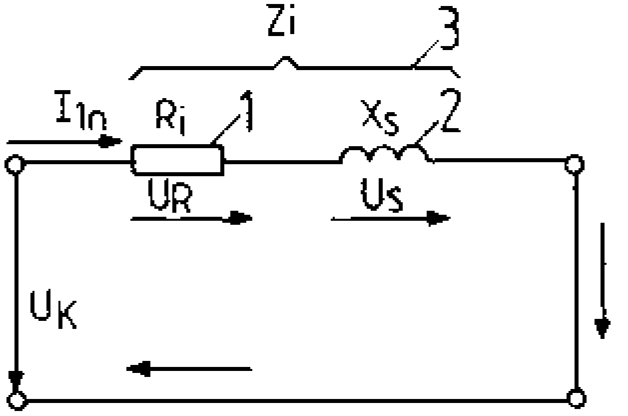
Transformers shall not be driven by voltages greater than the rated voltage $U_{1n}$.



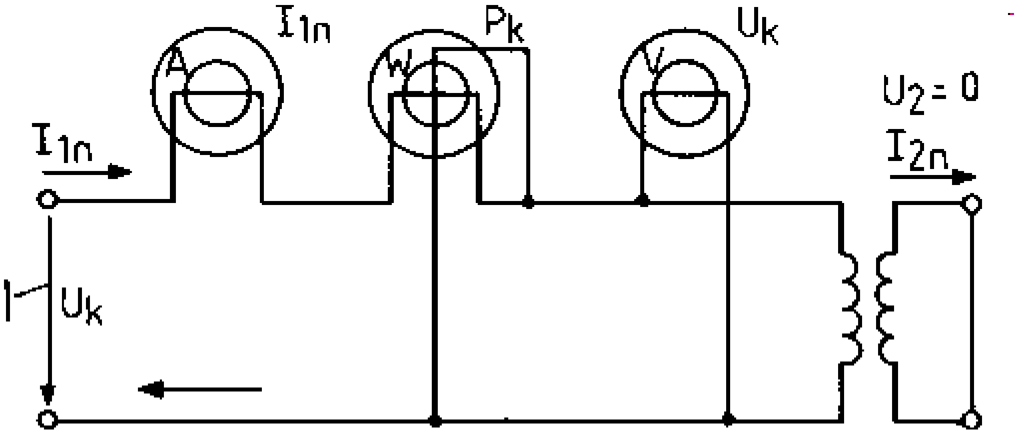
# Short-circuit behaviour

- Short-circuit curves
- Secondary current $I_2$ increases if load resistance is decreased. Where $Z_a = 0$ the transformer has been short-circuited.
- Primary circuit $U_1$ is applied
  $I_K$ flows Secondary circuit $Z_a = 0$
  $U_2 = 0$ Short-circuit voltage
- The short-circuited transformer can be replaced by resistor $Z_1$ which corresponds to the transformer internal resistor





# Short-circuit behaviour



The relative short-circuit voltage $u_K$ in % is determined by the following equation:

$$u_K = \frac{U_K}{U_{1n}} 100\%$$

The relative short-circuit voltage is, on average, 2 to 10% of input rated voltage ($U_{1n}$) in mains transformers

# Short-circuit behaviour

Short-circuit losses (winding losses)

- In the short-circuit experiment a power meter indicates short-circuit losses as the primary and secondary rated currents generate winding losses. The iron core is only slightly magnetised by the applied short-circuit voltage

$$U_K << U_1$$

- The winding losses can be metered during the short-circuit experiment. They are dependent on the load current ($P_{VW} = I^2R$).

# Loaded voltage behaviour

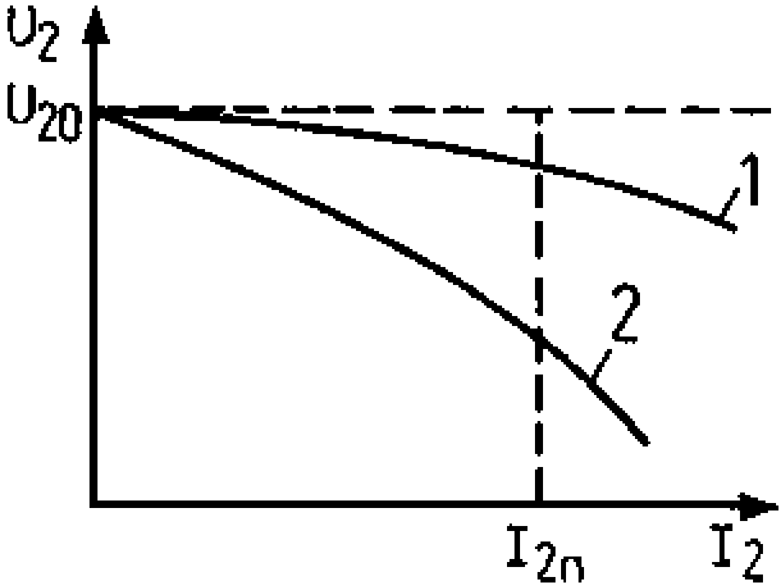
- In contrast to operational idling, during loading the secondary circuit is closed through an external resistance $Z_a$. Secondary current $I_2$ flows. According to the energy conservation law the transformer must also take up commensurate primary power, thus a primary current $I_1$ also flows.

- Primary circuit $U_1$ is applied

  $I_1 > I_0$ Secondary circuit $Z_a < \infty$
  $I_2 > 0$
  $U_2 < U_{20}$

# Loaded voltage behaviour

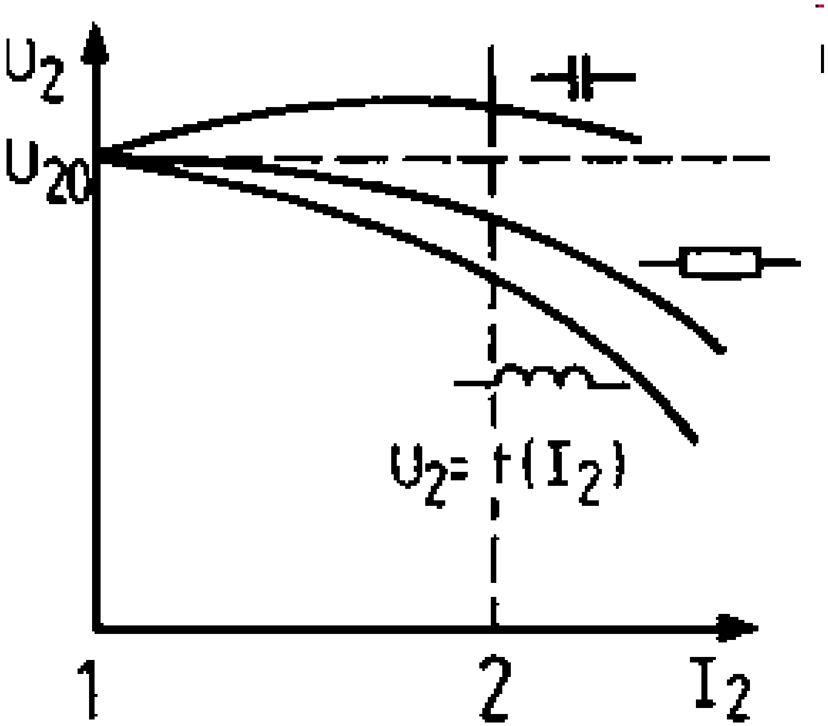
- Voltage curve $U_2 = f(I_2)$
- As the curve in figure shows, terminal voltage $U_2$ decreases during loading.



1 $u_K$ small, 2 $u_K$ big

# Loaded voltage behaviour

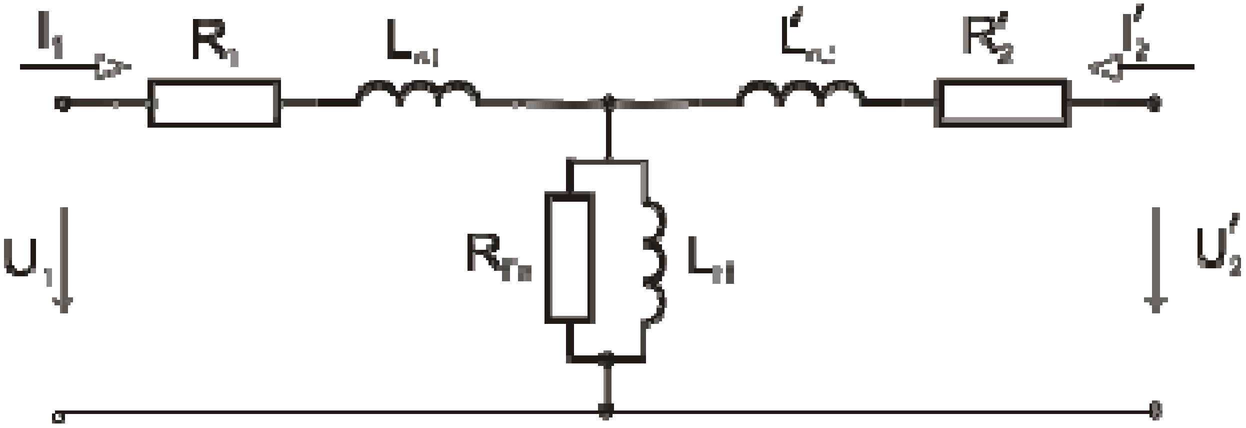
Secondary terminal voltage depending on the degree and nature of loading



1 Idling
2 Rated load

The output voltage of a transformer depends on the
- degree of load current $I_2$
- the magnitude of relative short-circuit voltage
- the nature of the load (ohmic, inductive or capacitive).

# Full equivalent circuit diagram



$U'_2 = p.U_2$ $L'_2 = p^2.L_2$

$I'_2 = 1/p.I_2$ $R'_2 = p^2.R_2$