Summary of recent results on design and application of signal shapers

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Abstract—This article provides summary of our previous work and applications of delay based signal shapers, which are widely used for vibration control of light weight flexible structure, like robotics manipulators, cranes, or another application in vibration control. Next to the analysis and application of classical signal shapers with lumped delays, completely original shapers with distributed delays are proposed. The shaper design methods are first demonstrated on a case study of flexible airplane, performed in the framework of the 7FP EU project ACFA2020. Feed-forward shapers compensator are used as smart filter of pilot commands as an alternative to classical Butterworth or Chebyshev filters. Further applications of signal shapers are shown on laboratory experiments portal crane and a servo with a flexible link.

Index Terms—Input shaping, Delay based input shapers, Lumped delay, Distributed delay, vibration control ...

I. INTRODUCTION

Input shaping is a feed-forward control technique for reducing vibrations in computer controlled flexible machines. Speaking in broad terms, the method works by creating a command signal that cancels its own vibration. That is, vibration caused by the first portion of the command signal (in time domain) is canceled by vibrations induced by the rest of the command. Input shaping is implemented by convolving a sequence of impulses, defining the input shaper, with the reference signal (step, or any other). The shaped command that results from the convolution is then used to drive the system. If the impulses defining the shaper are arranged in a smart way, the flexible system will respond without vibration to the reference command.

Significant filtering features of simple time-delay shapers were first reported by [5], and an application for effective manipulation and control of flexible systems was immediately proposed. The shaper is used to filter (shape) the reference signal carefully so that it, on one hand, does not contain frequencies of significant flexible modes which are therefore not excited, and, on the other hand, retains responsiveness of the system’s response. Next, Singer and Seering [6] and [9] re-visited the concept of delay-based signal shapers. They developed alternative methodology and time-domain formulas for the Smith’s posicast [5], giving rise to a new modification with improved robustness, the zero-vibration-derivative (ZVD) shaper and extra insensitive shaper (EI) [13]. Robustness analysis of classical shapers is discussed [6] and also in a recent paper [10].

II. CLASSICAL SIGNAL SHAPERS AND THEIR DESIGN

Singer and Seering’s approach is based on analysis of the response of a second order undamped system (1) to an impulse sequence by means of the vibration ratio given by (2)

\[ G(s) = \frac{\omega^2}{s^2 + 2\xi\omega s + \omega^2} \]  

\[ V(\xi, \omega) = e^{-\xi\omega t_1} \sqrt{[C(\xi, \omega)]^2 + [S(\xi, \omega)]^2} \]  

\[ C(\xi, \omega) = \sum_{i=1}^{n} A_i e^{\xi\omega t_i} \cos(\omega \sqrt{1 - \xi^2} t_i) \]  

\[ S(\xi, \omega) = \sum_{i=1}^{n} A_i e^{\xi\omega t_i} \sin(\omega \sqrt{1 - \xi^2} t_i) \]

where \(\xi\) is the damping and \(\omega\) is the natural frequency of the system. \(A_i\) is the amplitude, \(t_i\) is trigger time of one of the impulses defining the shaper, \(n\) is the number of these impulses. The vibration equation (2) yields the trigger times and amplitudes (3) of each impulse for zero vibration response of the second order system in the shortest possible time.
\[
\begin{bmatrix}
A_i \\
t_i
\end{bmatrix} = \begin{bmatrix}
\frac{1}{1+K} & \frac{K}{2T_d} \\
0 & \frac{1}{1+K/T_d}
\end{bmatrix}, A_i > 0, \sum_{i=1}^{n} A_i = 1
\]  

(3)

where \( K \) is given by (4).

\[ K = e^{\frac{\pi}{\sqrt{1-\beta}}} \]  

(4)

So far, the results correspond to classical posicast. Singer and Seering’s formulation however allows adding an extra equation (5) which gives rise to improved robustness w.r.t. model uncertainty. Thus obtained ZVD shaper features weaker dependency of performance on uncertain or drifting frequency of the underlying flexible mode. Resulting shaper is defined by three impulses (compared to just two for posicast or ZV shaper) defined by (6).

\[
\frac{\partial V(\xi, \omega)}{\partial \omega} = 0
\]  

(5)

\[
\begin{bmatrix}
A_i \\
t_i
\end{bmatrix} = \begin{bmatrix}
\frac{1}{1+K} & \frac{1+V_i}{4T_d} \\
0 & \frac{1}{1+K/T_d}
\end{bmatrix}
\]  

(6)

Further development of the vibration ratio concept (2) led the authors to yet another robust variant - extra insensitive shaper (EI, [13]) , representing a trade-off between robustness and nominal performance. If one admits some tolerable low level of vibration (7) for the nominal system model, equation (2) can be modified accordingly. Resulting EI shaper is slightly detuned for the targeted mode with parameters given by (8) (and hence does not perfectly suppress related vibrations), though, for a wider considered frequency band, its performance is superior in average compared to ZV or ZVD.

\[ V_t(\xi, \omega) = 5\% \]  

(7)

\[
\begin{bmatrix}
A_i \\
t_i
\end{bmatrix} = \begin{bmatrix}
\frac{1+V_i}{4} \\
\frac{1}{T_d}
\end{bmatrix}
\]  

(8)

In the application of a designed signal shaper, which is in general in a form of difference equation

\[ v(t) = \sum_{i=1}^{N} A_i w(t - t_i) \]  

(9)

where \( w \) and \( v \) are the shaper input and output, respectively, the shaper is linked to the system as shown in Fig. 1.

III. REMARK ON SPECTRAL FEATURES OF SHAPERS WITH LUMPED DELAY

Consider a ZV shaper [6], [9]

\[ S_{ZV}(s) = A + (1 - A)e^{-\tau s}, \]  

(10)

where \( A \in \mathbb{R}^+, A < 1 \). It is easy to show that the zeros of the function (10) are given as follows

\[ s_k = -\frac{1}{\tau} \ln \frac{A}{1-A} \pm j\frac{\pi}{\tau}(2k + 1), k = 0, 1, ..., \infty. \]  

(11)

As can be seen, (11) constitute a chain of zeros that is parallel to the imaginary axis. As it results from the argument of the logarithm in (11), \( \frac{A}{1-A} \geq 1 \) in order to have the negative real part of the zeros. Thus, the parameter \( A \in \left( \frac{1}{2}, 1 \right] \).

The aim of the ZV shaper (10) is to compensate the pole \( r_{1,2} = -\beta \pm j\Omega \) of a system that is in a serial connection with the shaper as shown in Fig. 1. Placing the dominant zero \( s_{1,2} \) of (11) at the position of \( r_{1,2} \), we obtain

\[ \frac{1}{\tau} \ln \frac{A}{1-A} = \beta, \]  

(12)

\[ \pi \frac{\tau}{\Omega} = \omega. \]  

(13)

From (12)-(13) the parameters of the shaper result as

\[ A = \frac{e^{\pi \tau}}{1 + e^{\pi \tau}}, \]  

(14)

\[ \tau = \frac{\pi \Omega}{\omega}. \]  

(15)

IV. SHAPERS WITH DISTRIBUTED DELAY

Following the results presented in [1] and [2], distributed-delay ZV shaper (or DZV shaper) - is formulated in this section. Using the distributed delay instead of the lumped delay, we define the DZV shaper as follows

\[ S_{DZV}(s) = B + (1 - B)e^{-(s-A)/\rho} = \frac{B(s+1-\beta)e^{-(s-A)/\rho}}{\rho}, \]  

(16)

where \( B \in \mathbb{R}^+, B < 1 \). As can be seen, the transfer function of the shaper has a pole at the origin of the complex plane. However, substituting \( s = 0 \) to the numerator of (16), we can see that the transfer function has a zero at the origin too. Thus, the pole and zero at the origin cancel each other. Rewriting the characteristic function

\[ B\dot{s} + (1 - B)(1 - e^{-s\rho}) = 0 \]  

(17)

into the form

\[ (\dot{s} + \frac{1 - B}{B})e^{s\rho + \frac{1-n}{\rho}} = \frac{1 - B}{B} e^{\frac{1-n}{\rho}}, \]  

(18)

we can compute the zeros of the DZV shaper numerically [4] or (16) using the Lambert W function as follows

\[ s_k = \frac{1}{\rho} \left( W \left( k \cdot \frac{1 - B}{B} e^{\frac{1-n}{\rho}} - \frac{1 - B}{B} \right) \right), k = 1, 2, ... \]  

(19)
Obviously, as \( W \left( 0, \frac{1 - B}{B} e^{1 - \vartheta} \right) = \frac{1 - B}{B}, \) \( s_1 = 0. \) In order to compensate a pole \( r_{1,2} = -\beta \pm j\Omega \) of a system that is in a serial connection with the DZV shaper, the parameters \( B \) and \( \vartheta \) need to be determined from the following equation

\[
-\beta \pm j\Omega = \frac{1}{\vartheta} \left( W \left( 1, \frac{1 - B}{B} e^{1 - \vartheta} \right) - \frac{1 - B}{B} \right). \tag{20}
\]

After some manipulation, we can determine \( \vartheta \) as the first nonzero root of the equation

\[
\Omega e^{\beta \vartheta} - \beta \sin \Omega \vartheta - \Omega \cos \Omega \vartheta = 0, \tag{21}
\]

which can easily be solved numerically. The second parameter is given by

\[
B = \frac{1 - e^{-\beta \vartheta} \cos \Omega \vartheta}{\beta - 1 + e^{-\beta \vartheta} \cos \Omega \vartheta}. \tag{22}
\]

In [1] and [2], further details on spectral features are provided. Next, the frequency response characteristics are studied in the papers. It is shown that the shapers with distributed delays have much better features in this aspect compared to shapers with lumped delays.

\[\text{V. Robustness}\]

Robustness of all introduced shapers in general, can be visualized by the sensitivity curves [10],[12]. All classical shapers (ZV,ZVD,EI) have well known properties of sensitivity function, given by equation (2). Mainly focus is on properties of the new one DZV and basic ZV shaper for following example.

Vibration compensation of the second order system, where compensated poles are placed at \( r_{1,2} = -\beta \pm j\Omega, \) where \( \beta = 2 \) and \( \Omega = 20 \) is nominal case used for shapers design. Parameters of the shaper for that system are ZV(\( A = 0.5783, \tau = 0.1579(s) \)) and DZV(\( B = 0.2175, \vartheta = 0.2621(s) \)) Sensitivity curves in this example, see Fig. 2, which shows dependency of the vibration ratio on normalized frequency at the time \( t_s, \) where reference command reaches a desired value (Measured by value of step response at the time \( t_s \)). DZV shaper has non-symmetric shape and smaller vibration ratio on one side of the curve than ZV shaper which is for higher frequencies than setting of shaper frequency.

Bode plots of ZV and DZV shapers in this case are depicted in Fig. 3. One can see that the shapers feature notches at antiresonant frequencies \( \omega_i = (2i - 1)\pi/t_1, i = 1, 2, \ldots \) The antiresonant holes “depth” is \( 20\log_{10}|A_2 - (1 - A_2)| \) for ZV shaper (blue).Explanation is quite simple: input sinus of a given frequency is according to the delay term \( t_i \) decomposed into two sinus waveforms (with amplitude \( A_2 \) and \( A_1 = 1 - A_2 \)) and for the frequencies \((2i - 1)\pi/t_1\) these waveforms are shifted exactly at \( T_d/2. \) Therefore during addition of these sine waveforms we obtain minimal value of their amplitude.

Frequency responses of the shapers in Fig. 3 indicate quantitatively their respective robustness properties. ZV shaper arrives unsurprisingly as the most sensitive (least robust) one. In contrast, DZV is obviously much less sensitive to frequency up-shifts (red). This is useful in many applications where changes of physical parameters give rise to increased undesirable flexible modes frequencies.

The effect is further demonstrated in time domain by step responses on Fig. 4 and 5. The nominal (design) frequency is 20 rad/s and the 20 rad/s to 40 rad/s band is considered.

\[\text{VI. Case study: BWB aircraft}\]

Results of the previous chapters are further developed and applied for the case study of large flexible blended-wing-body aircraft [3]. Data come from the ongoing European project ACFA 2020. ACFA 2020 (Active Control for Flexible Aircraft, www.acfa2020.eu) is a collaborative research project funded by the European Commission under the seventh research framework programme (FP7). The project deals with innovative active control concepts for ultra efficient 2020 aircraft configurations like the blended wing body (BWB) aircraft. The Advisory Council for Aeronautics Research in Europe (ACARE) formulated the "ACARE vision 2020", which aims for 50% reduced fuel consumption and related CO2 emissions per passenger-kilometre and reduction of external noise. To meet these goals it is very important to minimize the environmental impact of air traffic but also of vital interest for the
Fig. 4. Robustness of ZV shaper. For controlled-system frequency range (20-40 rad/s).

Fig. 5. Robustness of DZV shaper. For controlled-system frequency range (20-40 rad/s).

The aircraft industry is being enabled for future growth. Blended Wing Body type aircraft configurations on Fig. 6 are seen as the most promising future concept to fulfill the ACARE vision 2020 goals because aircraft efficiency can be dramatically increased through minimization of the wetted area and reducing structural load and vibration by active damping.

ACFA BWB longitudinal dynamics models used in this section contain 4 flexible modes, rigid-body dynamics, actuators and sensors models, and lag states (total order 30). Input command shapers will be attached to a feedback CAS, Fig. 7 (control augmentation system, pitch autopilot in particular, w/o active damping system), and effectively reduce vibrations caused by the pilots commands (pitch angle setpoint).

Fig. 7. Control augmentation system and input shaper ("Filter")

In Fig. 8 the flexible modes of the aircraft for the normal acceleration setpoint to (wingroot bending moment) channel is depicted in blue, for a set of mass cases, with clearly visible wing-bending and hull-bending modes. The red line corresponds to a ZV-ZV two-modes shaper, green line to its Pade approximation of total order 8. Performance of the Pade approximation of this relatively low order (compare to order 6 per notch as recommended in section 5) is clearly adequate and gives rise to comparable resulting performance compared to the delay-based shaper. Responsiveness of the overall control is demonstrated by Fig. 10 (pitch-rate step response) while the flexible vibration treatment is evident from Fig. 9 (wing root bending moment reaction).

Fig. 8. Wingroot bending moment frequency response

VII. experimental verification

Two related experiments were conducted and are reported further, proving the findings of the paper and bringing up some additional subtle issues. The portal-crane is an example of a system where the underlying servomechanism is considerably faster than the targeted modes (sway of the load). In this case, the slower overall response of the DZV-shaped step reference, compared to the standard ZV version, appears therefore as rather remarkable. On the other hand, in the flexible-link manipulator case, the...
proximity of flexible mode to the servo’s bandwidth results in measured transient responses (in position) with almost identical settling time, due to the fact that the ramp-like DZV signal is considerably easier to follow for the involved position servo. Increased robustness of the distributed-delay variant is visible very nicely in both cases. This section is adopted from preliminary version of[2], [1]

A. Portal crane

The 2-D crane on Fig. 11 is a laboratory model made of aluminum frame on which the crane head is located. The head is moving by two DC motor in two axes independently. Another DC motor is applied to alter the length of the rope carying the payload. Physical dimensions are - length= 1.60m width= 0.9m and high= 0.76m, see Fig. 11. • crane arm engine potential \( u_a \) [V] (actuating signal) • crane wind engine potential \( u_w \) [V] (actuating signal) • winding mechanism engine potetial \( u_c \) [V] (actuating signal) • crane arm position x [m] • crane wind position y [m] • 4 sensors of outer limits (DS 1-4) • 2 sensors of wire deviation\( \varphi_1 \) [°], \( \varphi_2 \) [°] Oscillation frequency of the load-sway mode is \( f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \), where L is the length of the cable and g is the gravity constant. The head-position servo reaches the closed-loop bandwidth \( \omega = 10rad/s \) an \( H_\infty \) mixed-sensitivity controller was designed, see [18], which is much faster then the oscillatory mode of \( \omega = 3.87rad/s \) and damping \( \zeta = 0.008 \).

Fig. 12 refers to the reference tracking performance for different feed-forward shapers. Although the DZV variant exhibits clearly visible slower settling time in this case (on the negative side; compare to the flexible link experiment below though), robustness with respect to changing (increasing) length of the load-carying cable, see Fig.13 for ZV shaper and Fig.14 for DZV shaper, is increased many times (note that for the extreme cases, the ZV gives even rise to saturated sway-angle sensors, blue dotted line) and for nominal case on Fig. 15.

B. Flexible link model

The Quanser Inc. laboratory experiment “Flexible Link” [19] was investigated next, with a lighter homemade link (C, L=35cm) attached to the original rotating table (A) on Fig. 16 (strong coupling of the rigid-body (B, r=35cm) and flexible dynamics (C) appears with the original heavy metal strip, which is undesirable in our application)

Position - angle of the table - is measured and actuated by feedback servo (designed again using the mixed-sensitivity \( H_\infty \) approach, [18]). Vibrations of the light weight flexible structure are measured by a video camera and processed off-line using a appropriate video-
Fig. 13. Measured swing of the load with ZV shaper for different lengths of the cable (0.3m-0.6m)

Fig. 14. Measured swing of the load with DZV shaper for different lengths of the cable (0.3m-0.6m)

Fig. 15. Measured swing of the load for nominal length of the crane cable 0.6m. ZV - blue dashed, DZV - red solid.

Fig. 16. Flexible link experiment setup.

Fig. 17. Measured angular position, with ZV (blue dashed), DZV (red solid) shapers, and without a shaper (green dotted)

processing software. The positioning servomechanism features achievable bandwidth of $\omega = 19 \text{ rad/s}$ while the vibrations (first bending mode) lies at $\omega = 19.9 \text{ rad/s}$.

Fig. 17 refers to the reference tracking performance for different feed-forward shapers. Note that as the bandwidth of the servo is very close to the flexible vibrations, compared to the portal-crane example above, the overall response time is not dominated just by the shaped reference signal, but also by the ability of the positioning feedback loop to track the shaped step input. In this regard, the DZV ramp-like signal is much easier to follow, giving rise to fairly comparable overall transient times in this and similar cases.

Regarding the level of flexible-link vibrations, see Fig. 18 and Fig. 19. The green line stands for no shaper engaged, the blue one for the classical ZV shaper, and red for the proposed DZV variant. In the nominal case, Fig. 18, results for ZV and DZV shaper are comparable in fact. For the non-nominal case, though, with increased frequency of the structural vibrations, the DZV shaper preserves its performance, while the classical ZV shaper becomes worthless - 19.
The presented research has been supported by the Ministry of Education of the Czech Republic under the program KONTAKT II LH12066, by the Grant Agency of the Czech Technical University in Prague, grant No. SGS11/150/OHK2/3T/12 and by EC project ACFA 2020 - Active Control for Flexible 2020 Aircraft under No. 213321.

**References**


[19] www.quanser.com