Modelling of High-Speed Permanent Magnet Synchronous Motor

M. Novak

Abstract — This paper presents specific issues of modelling a high-speed permanent magnet motor (PMSM). Although the dynamic model of classic PMSM is well known, high-speed machines have theirs specifics like a large influence of bearing or air friction losses not covered by the standard model. Those are discussed in this paper together with experimental verification of the model. Finally suggestions for future improvements are given.

Keywords — Mathematical modelling, permanent magnet synchronous motor, high-speed motor

I. NOMENCLATURE

\[ v_x(t) \] — instantaneous voltage value in phase x [V]
\[ R_s \] — stator resistance [Ω]
\[ \Psi \] — flux linkage [Wb]
\[ i_x \] — instantaneous current value in phase x [A]
\[ \phi \] — rotor angle [rad]
\[ L \] — inductance
\[ \omega \] — electrical speed [rad/s]
\[ \omega_m \] — mechanical speed [rad/s]
\[ T_e \] — electrical torque [Nm]
\[ p_b \] — number of pole-pairs [-]

II. INTRODUCTION

High speed permanent magnet motors became a hot topic in the last years. It is due to improvements in permanent magnets and motor design technology. The reason for creating smaller, high-speed machines is theirs much higher power density. As it is shown in [1] one large turbocompressor can be replaced with 16 compressors, each with a volume of 1/64 of the conventional compressor, which together has the same output power but requires only a quarter of the volume of the conventional compressor. The diameter of the small units would be ¼ of the original one and the rotational speed would therefore increase by a factor of at least 4.

This work was supported in part by the Czech ministry of education, youth and sport grant no. MSM6840770035 “The Development of Environmental - Friendly Decentralized Power Engineering”, internal CTU Grant “Development of measuring, simulation and experimental methods with focus on non-traditional energy source” and support of EU Regional Development Fund in OP R&D for Innovations (OP VaVpI) and Ministry for Education, Czech Republic, project # CZ.1.05/2.1.00/03.0125 Acquisition of Technology for Vehicle Center of Sustainable Mobility. This support is gratefully acknowledged.

M. Novak is with Department of Instrumentation and Control Engineering, Faculty of Mechanical Engineering, Czech Technical University in Prague, Prague, Czech republic (e-mail: Martin.Novak2@fs.cvut.cz).

Downscaling of a macro turbomachine for constant specific speed and lower volume flow therefore leads to an increase in rotational speed. The overall volume of the electrical machines in the example above is also ¼ of the original one.

From this comes the idea to use high speed machines also in other applications with size constraints. As it was shown in [2] where the target maximum speed of the motor is 240,000rpm, rated output is 5kW and the stator diameter is 60mm, core stuck size is 40mm and rotor diameter is only 20mm.

The paper is organized as follows. First a description of a standard PMSM model is given, that specifics of high-speed model are discussed. Follows the identification of real high-speed motor parameters and comparison with modeled data. Finally conclusions with suggestion for future model improvement are given.

III. PMSM MODEL

For the purposes of future FPGA controller implementation and verification we have created a Simulink model of PMSM. The model was used during the designing phase of the controller for algorithm verification, range checking, parameter setting etc. The model is based on the information published in [3] - [7].

The PMSM model is based on the electrical properties of the stator windings and on theirs interaction with the rotor. The electrical dynamic equations of the phase voltages \( v_a, v_b, v_c \) are:

\[ v_a(t) = R_s + \frac{d\lambda_a}{dt} \] (1)
\[ v_b(t) = R_s + \frac{d\lambda_b}{dt} \] (2)
\[ v_c(t) = R_s + \frac{d\lambda_c}{dt} \] (3)

where \( R_s \) is PMSM stator winding resistance and \( \lambda_a, \lambda_b, \lambda_c \) are corresponding flux linkages for phase a,b,c.

Under the assumption that mutual inductances \( L_{ab} = L_{ba} \) and peak flux linkage of the permanent magnet is \( \lambda_{im} \) (with components a, b, c) the flux linkages \( \lambda_a, \lambda_b, \lambda_c \) in (1) - are (3)

\[ \lambda_a = L_{aa}^i + L_{ab}^i \] (4)
\[ \lambda_b = L_{bb}^i + L_{bc}^i \] (5)
\[ \lambda_c = L_{cc}^i + L_{ba}^i \] (6)

Flux linkages, voltages and currents can now be transformed from the three phase stator coordinate system.
to the rotor fixed dq coordinate system using transformation matrices [6]. The symbol $S$ in those matrices represents voltages, fluxes or currents - Park transformation

$$
\begin{bmatrix}
S_q
\end{bmatrix} =
\begin{bmatrix}
\cos(\Theta) & \sin(\Theta) & 1
\end{bmatrix}
\begin{bmatrix}
S_d
\end{bmatrix}
$$

(7)

where $S_0$ is a zero vector, in a balanced three phase system $S_a + S_b + S_c = 0$

The inverse transformation (inverse Park transformation) is

$$
\begin{bmatrix}
S_a
\end{bmatrix} =
\begin{bmatrix}
\cos(\Theta) & \sin(\Theta) & 1
\end{bmatrix}
\begin{bmatrix}
S_q
\end{bmatrix}
$$

(8)

Using transformations (7) and (8) the voltage equations in dq reference frame becomes

$$
v_q = R_i q + \frac{d\lambda_d}{dt} + \omega\lambda_q
$$

(9)

$$
v_d = R_i d + \frac{d\lambda_d}{dt} - \omega\lambda_q
$$

(10)

$$
\lambda_q = L_q i_q
$$

(11)

$$
\lambda_d = L_d i_d + \lambda_m
$$

(12)

where $\omega$ is electrical speed and $\lambda_m$ is the flux linkage of permanent magnets.

This model is shown in an equivalent circuit diagram on Fig 1.

The electric torque produced by the PMSM is

$$
T_e = \frac{3}{2} \left[ v_q i_q + v_d i_d \right] = \frac{3}{2} p_p \left[ \lambda_a i_q + (L_d - L_q) i_d i_q \right]
$$

(13)

The above described electrical model is completed with a mechanical model

$$
T_e = T_L + \frac{J}{p_p} \frac{d\omega}{dt} + \frac{B}{p_p} \omega + \frac{K_{air~friction}}{p_p} \omega^2
$$

(14)

where $T_L$ is torque caused by the load, $J$ is moment of inertia, $B$ is damping constant (friction), $K_{air~friction}$ is damping caused by air friction and $p_p$ is number of pole-pairs.

Fig 1 - dq axis PMSM equivalent circuit diagram

Rotor mechanical speed and rotor angle is

$$
\omega_m = \frac{\omega}{p_p} \Theta(i) = \int \omega dt
$$

(15)

Hence the space state model of PMSM is

$$
\frac{di_d}{dt} = -\frac{R}{L_d} i_d + \frac{L_q}{L_d} i_q \omega + \frac{1}{L_d} v_d
$$

(16)

$$
\frac{di_q}{dt} = -\frac{R}{L_q} i_q - \frac{L_d}{L_q} i_d \omega + \frac{1}{L_q} v_q - \frac{\lambda_m}{L_q} \omega
$$

(17)

$$
\frac{d\omega}{dt} = \frac{3}{2} \frac{p_p^2}{J} \lambda_a i_q + \frac{3}{2} \frac{p_p^2}{J} (L_d - L_q) i_d i_q - \frac{p_p B}{J} \omega - \frac{p_p K_{air~friction}}{J} \omega^2 - \frac{p_p}{J} T_L
$$

(18)

For simplicity, the space state model can also be rewritten in a matrix form

$$
\frac{dX}{dt} = A \cdot X + Bu + N_1 \cdot X \cdot \omega + X^T \cdot N_2 \cdot X
$$

(19)

where $A$ is system matrix, $B$ is input matrix, $N_1$ and $N_2$ are nonlinear coupling matrices.
Based on (20) – (23) PMSM model was created.

IV. REAL MODEL PARAMETER IDENTIFICATION

To set correctly the PMSM model it was necessary to estimate some model parameters. Some of them could be determined directly from the manufacturer’s data while others required some experiments.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>PMSM PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor type:</td>
<td>2AML406B-090-10-170</td>
</tr>
<tr>
<td>Manufacturer:</td>
<td>VUES Brno</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>560 V</td>
</tr>
<tr>
<td>$I_{dc,m}$</td>
<td>11 A</td>
</tr>
<tr>
<td>$L_{dc},m$</td>
<td>1.2 Nm</td>
</tr>
<tr>
<td>$n_o$</td>
<td>25 000 min⁻¹</td>
</tr>
<tr>
<td>$n_{max}$</td>
<td>25 000 min⁻¹</td>
</tr>
<tr>
<td>$K_E = 7.3$ V/kRPM</td>
<td></td>
</tr>
</tbody>
</table>

To estimate other motor parameters two different methods were used and the results compared.

Method one consisted of a direct measurement with LCRG Meter Tesla BM591 where both winding resistance and inductance was measured. The disadvantage of this method is that the measuring current is not known and that inductance will be measured with a small unknown current. As inductance is a function of current, it is expected that the real inductance value for nominal current will be higher.

In method two, resistance was measured with Ohm’s method with a Diametral Q130R50D power supply, 2x Pro’s kit digital multimeter MT1232. The inductance in method two was measured from the time constant of a transient characteristics with oscilloscope GDS-806C with probe GTP-060A.

As stator inductance is a function of current, the measurement was done up to the nominal current and a little above it. The results are summarized on Fig 2. It can be seen that the inductance is changing some what with current as it was expected. It is rising with increasing current. For nominal current, the inductance is 1.1 mH. It is also obvious that the measurement has a relatively high error as the values are fluctuating. Nevertheless the trend is visible. The obtained results from both methods are the following

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>PMSM RESISTANCE AND INDUCTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct measurement with BM591</td>
<td>Indirect measurement</td>
</tr>
<tr>
<td>$R$ (mΩ)</td>
<td>310</td>
</tr>
<tr>
<td>$L$ (µH)</td>
<td>880</td>
</tr>
</tbody>
</table>

As can be seen from the comparison in the table above, the values obtained from both methods are similar but with a relatively high error. The error in stator winding resistance is large because the value of stator winding resistance is small and its size is comparable with the contact resistance in the circuit. The inductance values are comparable and it is expected that a more precise value is the one obtained with indirect measurement as nominal current 11 A is used here.

The permanent magnet flux linkage $\lambda_m$ is calculated from the back-emf constant $K_E = 7.3$ V/kRPM

$$\lambda_m = \frac{V_o}{\omega} \Rightarrow \lambda_m = 0.072 \ [Wb]$$

Motor’s moment of inertia was calculated from a motor startup on Fig 3. In this experiment where the motor was powered with a given current $I_q = 9$ A until field weakening started. The start time was 0.4 s and the reached speed was 572 Hz (34 340 min⁻¹). The startup current 9 A corresponds to torque 0.98 Nm. Considering the relatively high current and torque, mechanical losses were neglected in the calculations [8].
Moment of inertia is
\[
J = T \frac{\Delta t}{\Delta \omega} = 0.98 \cdot \frac{0.4}{2\pi \cdot 572} = 0.11 \cdot 10^{-3} \quad [\text{kg} \cdot \text{m}^2]
\]  \hspace{1cm} (25)

As can be seen from the model, air friction losses are considered. This is important for high-speed machines as the losses caused by air friction can have the size as friction losses in bearings.

According to [9], where speed was 500 000 RPM, friction losses caused by air friction were 8 W, where as bearing losses were 10 W for two bearings.

As the machine used for this research has maximal speed 42 000 RPM, it can be expected that air friction losses will be much lower. The reason for this is that according to [9] power loss caused by air friction is given
\[
P_{\text{f, air}} = c_f \rho_{\text{air}} \omega^3 r^4 l
\]  \hspace{1cm} (26)

Where \( c_f \) is friction coefficient, \( \rho_{\text{air}} \) is density of air at given temperature and pressure, \( \omega \) is rotor angular speed, \( r \) is rotor radius and \( l \) is rotor length.

Air friction torque is then
\[
T_{f, \text{air}} = c_f \rho_{\text{air}} \omega^3 r^4 l \div \omega
\]  \hspace{1cm} (27)

And therefore the friction torque is a function of \( \omega^2 \) as it is used in the model.

The friction coefficient itself is dependent on the size of air gap and air flow in the air gap given by Reynolds and Taylor numbers. Unfortunately, none of those parameters could not be measured or determined precisely analytically.

For this reason an attempt was made to at least estimate bearing and air friction with an experiment. It consists of accelerating the motor to maximal speed and turning off the inverter. The rotor will spin down naturally. The deceleration of the rotor is measured as a function of time.

From (18) it is obvious that when the motor is unpowered and unloaded, its mechanical speed will decrease with losses until a complete halt. This is described by the following dynamic equation
\[
\frac{d\omega}{dt} = -\frac{p_B}{J} \omega - \frac{p_p K_{\text{air, friction}}}{J} \omega^2
\]  \hspace{1cm} (28)

It has to be noted here that this equation does not represent all losses in the motor.

![Fig 3 - PMSM startup with Vdc = 500 V, \_Iqreq = 9 A](image)

![Fig 4 - Comparison between measured (blue, solid line) and modeled PMSM deceleration (red, dashed)](image)
The solution of (28) is
\[ \omega(t) = \frac{B \cdot \omega(0)}{B \cdot e^{\frac{-t}{T_m}} - K_{air\_friction} \cdot \omega(0) + K_{air\_friction} \cdot \omega(0) \cdot e^{\frac{-t}{T_m}}} \]  \hspace{1cm} (29)

Equation (29) was used to find coefficients \( B \) and \( K_{air\_friction} \) from experimentally measured PMSM rotor deceleration. The search was done with a least square method in Matlab. The comparison on Fig 4 is a best match that could be achieved by varying just parameters \( B \) and \( K_{air\_friction} \). As can be seen, the match is not very good. The reason is that other losses in the motor have been neglected.

One is the loss caused by eddy currents. As the permanent magnet is rotating, it induces currents to the stator windings and stator iron. Also there is an interaction between the permanent magnet and those currents. Stator core losses could be determined by the Steinmetz equation [10],[11]
\[ P_{core} = C_m \cdot f^\alpha \cdot B_m^\beta \]  \hspace{1cm} (30)

Where \( C_m, \alpha \) and \( \beta \) are material constant, \( B_m \) peak flux density and \( f \) is frequency of the current. It can be seen that losses are a function of frequency i.e. rotational speed. The function is non linear. Unfortunately this calculation is impossible as the stator material is not known for our motor.

Another loss that has been neglected is the interaction between the permanent magnet and stator iron. In other words, the permanent magnet is attracted to the stator iron and is braked by it. This is visible when the rotor is turned by a bare hand. It takes then preferably one of four positions. This effect seems to be significant for lower rotor speeds.

Considering those simplification, the presented fit can be considered an approximate model. The model parameters determined in this chapter are summarized in Table III.

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>PMSM MODEL PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model parameter</td>
<td>Value and units</td>
</tr>
<tr>
<td>( R ) (mΩ)</td>
<td>260 [mΩ]</td>
</tr>
<tr>
<td>( L_1 ) (µH)</td>
<td>1100 [µH]</td>
</tr>
<tr>
<td>( \lambda_m )</td>
<td>0.072 [Wb]</td>
</tr>
<tr>
<td>( J )</td>
<td>0.11 \times 10^{-3} [kgm^2]</td>
</tr>
<tr>
<td>( B )</td>
<td>8.2 \times 10^{-10} [Nrad/s]</td>
</tr>
<tr>
<td>( K_{air_friction} )</td>
<td>1,3 \times 10^{10} [Nrad^2/s]</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

As it can be seen from the comparison between the measured deceleration and modeled one, there is a relatively large difference. For high-speed motors more parameters that just the ones present in a standard PMSM model have to be considered. It is the loss caused by the eddy currents caused by the permanent magnet rotation and the interaction between permanent magnet and stator iron that comes into play for small speeds. All those should be considered in future model development. However it can be expected that if a system with controller is used also some other components like the inverter has to be modeled to achieve a good agreement between the model and reality.

REFERENCES


